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# Prediction of performance of granular filters for water treatment

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PREDICTION OF PERFORMANCE OF GRANULAR FILTERS  
FOR WATER TREATMENT

by

Kou-ying Hsiung

A Dissertation Submitted to the  
Graduate Faculty in Partial Fulfillment of  
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DOCTOR OF PHILOSOPHY

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Approved:

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## LIST OF SYMBOLS

$B_0, B_1, B_2$	coefficients in regression equation
$C$	concentration of suspended particles at any depth and time
$C_0$	concentration of particles in suspension liquid entering the filter
$\bar{C}_0$	average influent concentration
$C_1, C_2, C_3, C_4, C_5, C_6$	constants in probability distribution functions
$d$	mean grain size of filter media
$d_{eq}$	equivalent mean grain size of graded media for the prediction of removal efficiency, defined as $1/2(P_{10}+P_{60})$
$d'_{eq}$	equivalent mean grain size of graded media for the prediction of removal efficiency, defined as $p_i d_i / 100$
$d_i$	geometric mean size of adjacent sieves
$f_0$	porosity of clean filter media
$f(X)$	probability function, $f(X) = 1$ , for all $X$ , if $X$ is a discrete variable; or $f(X)dX = 1$ , if $U$ is a continuous variable
$G$	a grouped term defined as $Q^{a_1} d^{a_2} t$
$H_0$	head-loss at the beginning of filtration through clean filter
$H_t$	head-loss during filtration at time $t$
$k$	number of "successes" in $n$ trials, a random variable
$k_1$	a filter coefficient
$k_2$	a proportionality factor in Ives' equation
$k_3$	a constant in Ives' equation

$k_4$	a constant in Ives' equation depending on the packing of the grains
$k_5, k_6$	indices in Ives' equation
$K_a$	breakthrough index, minimum value
$K_b$	breakthrough index, maximum value
$L$	distance into filter from inlet surface
$L_a$	depth of floc penetration, weakest flocculation
$L_b$	depth of floc penetration, strongest flocculation
$M$	total number of particles in the influent suspension
$m$	number of particles deposited between $L$ and $L + L$
$N$	a parameter in Poisson distribution, which is equal to $np$
$n$	total number of independent trials
$p$	probability of "success" in Bernoulli trial
$P_{10}, P_{30}, P_{50}, P_{60}$	size of grain in mm such that 10%, 30%, 50% and 60% respectively of the particles by weight are smaller than the stated size
$P_c$	cumulative probability, for a continuous random variable $f(X)dX = dP_c$
$P_i$	weight fraction of sample (%) separated between adjacent sieves
$P(k; n, p,)$	probability of resulting in exactly $k$ "success" in $n$ trials when the probability of a success at each trial is $p$
$P(L; rt, p,)$	probability of a particle moving to $L$ at time $t$

$P(z; n, p,)$	probability of a particle arriving at the position $z$ from the origin where the random walk starts within $n$ steps when the probability of resulting a downward displacement at each step is $p$
$Q$	filtration rate per unit area of filter, or approach velocity of filtration
$q$	probability of "failure" in Bernoulli trial
$R$	a grouped term defined as $d^{b_1}(H_t - H_0)/Q^{b_2}C_0^{b_3}$
$r$	average rate of particle movement within the filter bed in an analogy of random walk model
$S$	surge amplitude
$t$	filtration time
$U$	deposit index, derived from the variate of chi-square distribution based on 1 hr = 1 degree of freedom
$U_{\frac{1}{2}}$	deposit index, based on 1/2 hr = 1 degree of freedom
$U_{100}$	deposit index, based on 100 hr = 1 degree of freedom
$v$	degree of freedom in chi-square distribution
$X$	the variate, which is the random variable in the probability distribution
$\hat{Y}$	dependent variable in multiple regression
$Z$	independent variable in multiple regression
$z$	distance from the origin of random walk
$\Gamma$	symbol of gamma function
$\lambda$	impediment modulus
$\lambda_0$	impediment modulus for clean filter media
$\sigma$	specific deposit, or the volume of deposited material per unit of filter volume

$\sigma_u$

specific deposit at equilibrium in filter,  
i.e. ultimate specific deposit

## I. INTRODUCTION

Filtration is the operation in which a fluid and heterogeneous mixture of particles of solid are separated by a filter medium which permits the flow of the fluid but retains the particles of solids. In all types of filtration the mixture flows as a result of some driving force, i.e. gravity, pressure (or vacuum) or centrifugal force. In a municipal water treatment plant the most common filter medium is sand and the driving force is usually provided by gravity. Two types of sand filters are commonly used: one is the slow sand filter in which the removal of suspended matter is predominantly on the top layer. Another is the rapid sand filter, in which the sand is coarser and the flow rate is higher than is used in slow filters; thus depth removal also becomes significant.

Since the historic work of Fuller (1898), rapid sand filtration has become widely accepted. However, because of the lack of adequate understanding of the mechanism of filtration, the filter design has been limited by conservative empirical standards for selection of the media size, filtration rate, and terminal loss of head without much regard to the pretreatment and the character of influent which will be filtered. Experience has taught that a rapid sand filter having 2 ft of 0.5 mm sand, operating to 8 ft loss of head at a filtration rate of 2 gpm per square foot of filter area,

usually will provide safe water. Although the modern trend has been toward the use of higher rates and coarser sands, the basic design criteria have not been changed much. Sometimes, a filter designed and built on these criteria, does not give a satisfactory effluent or gives a very short filter run. The failure is often attributed to the unusual character of the raw water. Even though filters may produce good effluents, no information about the inherent factor of safety has been known in the design. Therefore, it has been a great concern to sanitary engineers to find a way to predict the performance of filters during the design stage.

Much work has been done in an attempt to elucidate the mechanism of filtration and to set up mathematical expressions to describe the changes which will take place within the filter during the filtration process. More understanding about the mechanism of filtration and the clogging process has been gained from both mathematical analyses and functional investigations.

In developing the theory of filtration, the well-known Kozeny equation relating to fluid flow through porous media has often been used. Kozeny considered the porous media as parallel capillaries in his derivation. Therefore, most filtration equations have been based on a model of complete orderliness of the filter media. On the contrary, a model of complete randomness relating to the hydrodynamics of the flow through porous media has been proposed by Scheidegger (1957)



and others. Since the actual hydrodynamic condition of the filter is likely to be somewhere between those two models, the random model will be at least as good as, if not better than, the other one. Regarding the removal of suspended matter by granular filters, Stein (1940) suggested that chance contact was the principal mechanism which has been reemphasized recently by Craft (1966), who thought that further development of this idea may lead to fruitful results. Many other possible mechanisms involved in filtration such as Brownian motion, straining, sorption, diffusion, inertial impingement and flocculation have also been described using basic probability theory. Therefore, it seems logical to consider the clogging process in filters as probabilistic in nature. Such a mathematical approach to the phenomena of filtration as a random process has been proposed by Litwiniszyn (1963).

In this study, a new parameter derived from a probability function is proposed for prediction of filter performance by graphical extrapolation of the data obtained from observations on small, shallow depth sand filters. Using this new parameter, the effects of the principal filtration variables on filter performance have been evaluated. Adequate information of this type may facilitate a rational design of sand filters.

## II. WORK OF OTHER INVESTIGATORS ON FILTER PERFORMANCE

Previous work on sand filter performance done by other investigators will be briefly reviewed in the following sections.

### A. Basic Mechanisms

It has been suggested that many possible mechanisms are involved in the filtration process for the removal of suspended particles from the fluid. These mechanisms can be divided into two categories: a transport mechanism and an attachment mechanism.

Particles in the flow stream must be first transported to the vicinity of the grain surfaces. Principal mechanisms, or driving forces, in this category may include the following:

- a. Straining
- b. Gravity
- c. Interception
- d. Hydrodynamics
- e. Diffusion
- f. Inertia
- g. Brownian motion

When particles are close enough to the grain surfaces, whether or not they can be separated from the flow stream is governed by the attachment mechanism. The two principal mechanisms are believed to be the result of:

- a. Van der Waals forces

b. Electrical double-layer interaction

Baylis (1937) and Hall (1957) have postulated that small particles can be removed by straining in the crevices where two sand grains touch each other. Straining effects will be generally operative if there are relatively coarse particles present in the suspension. But practically only the small unsettleable particles will reach the filter of a well operated plant; thus, the straining effects become insignificant as concluded by Stanley (1955), Cleasby and Baumann (1962), Hunter and Alexander (1963) and Ives (1967). A heavy surface mat on a porous medium may signify the importance of straining effects; however, such surface removal should be avoided in normal operation because short filter runs result.

Hazen (1904), Fair and Geyer (1954), Stanley (1955) and Hall (1957) have postulated the sedimentation mechanism. Ives (1967) has elaborated the concept of a gravitational velocity vector deflecting particles from the streamlines to a grain surface; this has been demonstrated in both downflow and upflow filtration where particles are collected preferentially on the tops of the sand grains. However, the importance of such a size-dependent removal mechanism has been discounted as a major mechanism by Stein (1940) and Cleasby and Baumann (1962).

The interception mechanism postulated by Stein (1940), Grace (1956) and Ives (1967) assumes that suspended particles will be forced close to a grain by the converging fluid

streamlines.

Hydrodynamic effects have been emphasized by Ives (1967). Particles in suspension may be brought into contact with a grain surface by hydrodynamic effects such as asymmetry forces on the particles due to shear flow and particle asymmetry.

Hunter and Alexander (1963) have shown that clay particles will diffuse across streamlines in the presence of a concentration gradient. Ives (1967) states that for particles of the order of one micron or less in size, diffusion may be a significant mechanism.

Inertia and Brownian motion are generally considered to be insignificant in rapid sand filtration.

It has been believed that surface forces between the grain and the suspended particle will operate when a particle approaches a grain surface very closely as the result of any of the foregoing transport mechanisms. Van der Waals forces arise from molecular attraction and increase in intensity as the particles approach each other. Mackrle and Mackrle (1961) have contended that adhesion between suspended particles and filter media is controlled by Van der Waals forces. Stanley (1955) proposed that the suspended particles are most easily removed when their electrokinetic repelling forces are at a minimum. Cleasby and Baumann (1962) have suggested that electrokinetic forces are primarily responsible for the removal of the hydrous ferric oxide particles. Hunter and Alexander (1963) have found that deposition of negatively

charged kaolinite in the sand filter is substantially increased by a reversal of the electric charge on the silica surfaces from negative to positive by adsorption of cetyl trimethylammonium ion, suggesting direct adsorption of the negative clay particles onto the positive surfaces. Some chemical influences on filtration have been studied by O'Melia and Crapps (1964).

The various postulated mechanisms as summarized above may interact with one another. It seems impossible to evaluate each mechanism individually; only some predominant phenomena have been demonstrated experimentally.

#### B. Mathematical Analyses

The first attempt at a mathematical analysis on the theory of filtration was proposed by Iwasaki (1937), suggesting that: (a) filtration is a dynamic process, its action being dependent on depth of, and time in, the filter; (b) the removal of suspended particles through the depth of the filter is proportional to the concentration of particles; (c) the constant of this proportionality increases linearly with the amount of clogging, which is time dependent; (d) the material removed from suspension clogs the filter pores. Their mathematical expressions are as follows:

$$-\frac{\partial C}{\partial L} = \lambda C \quad \text{Eq 1}$$

$$-\frac{\partial C}{\partial L} = \frac{1}{Q} \frac{\partial \sigma}{\partial t} \quad \text{Eq 2}$$

$$\lambda = \lambda_0 + k_1 \sigma \quad \text{Eq 3}$$

Where:

C = concentration of suspended particles at any depth and time

L = depth of filter

$\lambda_0$  = impediment modulus for clean filter media

$\lambda$  = impediment modulus

$k_1$  = filter coefficient

$\sigma$  = specific deposit or the volume of deposited material per unit of filter volume

t = filtration time

Q = filtration rate, in volume of water filtered per unit area of filter, or approach velocity of filtration

Iwasaki's first equation states that the amount of suspended particles arrested in a lamina is proportional to the amount that goes in.

Iwasaki's second equation is a continuity equation which states that the decrease of the concentration through a lamina is equal to the increase of the deposit density therein; in other words, the matter is neither created nor destroyed in any region of the filter bed, and the increase of deposit density is fully accounted for by the reduction of suspended particles in the flow. Therefore, the model is applicable only to a stable inert and discrete matter, and self porosity of the suspended particles is not accounted for.

Iwasaki considered that the probability of impediment would increase in a manner similar to filling up the meshes of

sieves with the small particles in the raw water during filtration. He postulated that the impediment modulus would steadily increase as the filtration proceeds; in other words, the removal efficiency becomes steadily better and better.

Iwasaki's ideas have been generally accepted as reasonable working hypotheses, although modifications of his third equation have been made by others.

Stein (1940) was the first investigator who made such modification on the change of impediment modulus during the filtration process. He suggested that contact of suspended particles on the sand grains and the adherence of such particles was the principal agency affecting removal in a sand filter. Small particles which were intercepted by the sand grains tend to build coatings of floc on the grains. The growth of these films reduced the dimensions of the constrictions of the passageways and caused an increase in the rate of removal of suspended matter by a lamina of a filter. Such coatings in the passageways increased the intensities of the viscous shearing forces, and as filtration continued, these shearing forces began to inhibit the rate of removal. Further entrainment of suspended matter caused a rapid decline in the rate of removal. Eventually, if the shearing forces became great enough, the coatings on the grains would fail in shear and be dislodged and many of the passageways which remained open were then so large that the interception of suspended particles was impossible, and the rate of removal became

substantially zero. Stein elaborated on the mathematical theory of Iwasaki and developed equations for the time rate of removal of suspended matter in a rapid sand filter. The equations were applied to the actual experimental data of Eliassen (1941) and gave reasonable agreement with the experimental observations.

Hall (1957) attempted to determine the dependence of  $\lambda$  on  $\sigma$  assuming that interstitial straining and gravitational settling were predominant mechanisms. By suitable selection of arbitrary parameters involved in his description, he was able to describe the initial stages of filtration quite well.

Ives (1960a) developed a mathematical model following Iwasaki's and Stein's ideas. He modified Iwasaki's third equation with the following limitations:

- a. The suspended particles are discrete, homogeneous, unisize, denser than the fluid and about two order of magnitude smaller than the filter pores.
- b. The filter medium is granular, isotropic, homogeneous and unisize.
- c. The fluid is in laminar flow.

Ives further hypothesized that the particles in the flow were deflected by gravity from streamlines and any particles that were brought within range of the Van der Waals forces of the granular media, or existing deposits, would adhere to the surface exhibiting such forces. Ives justified that the change of impediment modulus as related to specific deposit



during early stage of a filtration run could be expressed by  $\lambda = \lambda_0 + k_1\sigma$  as suggested by Iwasaki. As more material was deposited in the filter, the rate of removal of suspended particles from the flow would be reduced principally due to the increase of interstitial velocity and decrease of specific surface area available for deposit. The change of interstitial velocity was equal to  $\left[ \frac{Q}{(f_0 - \sigma)} - \frac{Q}{f_0} \right]$  or  $\frac{Q\sigma}{\left[ f_0(f_0 - \sigma) \right]}$  and the change of specific surface area was approximately proportional to the specific deposit. The total reduction in impediment modulus was then proportional to the product of these two effects and equal to  $k_2 Q \sigma^2 / \left[ f_0(f_0 - \sigma) \right]$ .

Where:

$f_0$  = porosity of clean filter media

$k_2$  = proportionality factor

For a given filtration rate and filter arrangement,  $Q$  and  $f_0$  would be constant, and could be grouped with  $k$  into one constant  $k_3$  and the equation for the impediment modulus as related to specific deposit became:

$$\lambda = \lambda_0 + k_1\sigma - \frac{k_3\sigma^2}{(f_0 - \sigma)} \quad \text{Eq 4}$$

Ives in discussing the paper by Fox and Cleasby (1966), generalized the analysis of the impediment modulus based on a combined sphere and capillary filter model. He suggested that initially the filter bed could be represented by an assembly of individual spheres, but as the deposits became contiguous, side spaces were filled in and flow through channels would then approximate flow through capillaries. It was assumed

that the impediment modulus was directly proportional to the specific surface. The interstitial velocity was incorporated as another limiting factor, so that the general expression became:

$$\lambda/\lambda_0 = (1+k_4\sigma/f_0)^{k_5} (1-\sigma/f_0)^{k_6} (1-\sigma/\sigma_u) \quad \text{Eq 5}$$

Where:

$k_4, k_5, k_6$  = constants, dependent on operation and filter conditions

$\sigma_u$  = specific deposit at equilibrium in filter, i.e. ultimate specific deposit

In the special case when  $k_5 = k_6 = 1$ ,

$$\lambda/\lambda_0 = 1 + \text{constant}_1\sigma - \text{constant}_2\sigma^2 + \text{constant}_3\sigma^3 \quad \text{Eq 6}$$

Ives contended that equation 4 conformed to a special case of the general condition as expressed in equation 6 with the last term neglected. But Fox and Cleasby (1966) questioned the validity of the last term of equation 4 to represent the filtration of hydrous ferric oxides floc by uniform sand.

The chief difficulty in the use of equation 4 or 5 results when one attempts to determine the specific deposit, in terms of volume of suspended particles removed per unit filter volume. A conversion factor must be used to change the value of specific deposit, commonly measured in weight per unit volume to that of volume per unit volume. To develop such a conversion factor for a particular floc, information about floc density and the possible change of floc density after deposition is required. Ives' equations could be solved

by numerical analysis with aid of a digital computer, provided that the constants were determined experimentally. However, the applicability of Ives filtration equations depend largely on the accuracy of experimental determination of these constants and the calculation of specific deposits. Regardless of these difficulties, Ives' work has been by far the most satisfactory theoretical treatment, and has been extended to deal with filters containing graded media (Diaper and Ives, 1965). However, its practical application is still limited.

Ives (1961b) also cited the work by Shekhtman (1961) who developed a filtration theory based on arguments very similar to Iwasaki's. Shekhtman's propositions lead to a fairly straightforward mathematical equation showing that the filtrate quality and clogging depend on both filter depth and time of operation.

Mints (1960) developed a filtration theory based on the hypothesis that the filtration process consisted of a constant deposition in the filter pores together with a shearing away of existing deposits. Mints drew on Eliassen's experimental data to support his theory. However, the dislodging concept had been previously refuted experimently by Stanley (1955).

Mackrle and Mackrle (1961) developed a new theory of filtration based on the assumption that Van der Waals forces controlled the removal of suspended material in the pores of a filter. They related the Van der Waals forces between the filter grain and suspended particle to the hydrodynamic drag

on the particles, according to Stokes' equation.

Camp (1964) presented the results of the studies by Stein and analyzed the results of Eliassen's experiments. He developed an equation to express the hydraulic gradient at various depths and times in terms of the corresponding specific deposit, so that the specific deposit values could be computed from the hydraulic gradient data without using the measured filtrate concentration. Camp emphasized the importance of pilot plant study which should precede filter design and presented a graphical method for the prediction of removal based on pilot plant observations of head-loss.

Litwiniszyn (1962) interpreted the filter clogging process based on stochastic statistics. According to his hypothesis, for uni-size spherical particles the rate of clogging is proportional to the momentary number of free pores still susceptible to be blocked. He treated the filtration process as a particular probability problem assuming that the lower the probability of a particle in the suspension being seized by the pores of the porous media, the lower the number of non-clogged pores, i.e. when all pores are blocked, the probability becomes zero. Mathematical expressions were developed which stated that the concentration of deposited material within the bed would be equal to zero at the beginning of filtration and approached to the saturated condition when the filter run was sufficiently long.

### C. Functional Investigations

Baylis (1926) made the first report of detailed observation of filtration action. He observed through a pilot filter with a glass side the predominant surface removal. He noted that as the surface layer became clogged, small channels would break open to permit sediment to pass to a slightly lower layer where the water would fan out in the clean sand and suspended particles would again be removed. In his later paper (1937), investigation was made on the filter performance at the end of run for a given depth. He reported that the length of filter run was observed to vary inversely as the 1.2 to 2.0 power of the filtration rate, and approximately directly as the 2.15 power of the diameter of the effective size of sand grains. The increase of filtration rate caused a slight depreciation in the effluent quality.

Allen (1935) conducted experiments concerning the filtering properties of different grades of sand, in co-operation with workers at a number of other cities.

The term "critical depth" was used in his work and was defined as the minimum depth of a filtering medium which would deliver a clear effluent up to a loss of head of 8 ft under all conditions of a raw water. This was based on the concept that the properly treated water in passing through a filter would be clear after it reached the point of maximum depth of sediment penetration. The depth of this point below the surface of the sand was therefore the critical depth of the

bed.

Hudson (1938) presented a paper to emphasize the usefulness of small glass tube filters which had given results comparable to those obtained from large units. He also reported that length of filter run for ten different filter media varied as the 3.8 power of the porosity of the filtering material, indicating that angular materials would give longer filter runs than rounded ones.

Later (1948, 1958), he proposed an empirical parameter of filtrability called the "breakthrough index" which was expressed by  $Qd^3H_t/L$  or  $Qd^3H_t^{0.5}/L$  for conditions of weak flocculation or strong flocculation respectively. He concluded that under either condition of flocculation, the penetration of suspended matter into the bed seemed to be directly proportional to the filtration rate. He suggested that higher filtration rates could be used with proportionately thicker beds, finer sands, or lower head-losses, without impairing filtered-water quality.

Hudson (1963) developed two empirical equations to express the relationships between the variables governing the design of rapid sand filters as follows:

$$C = f(Q, d^3, f_o^3, H_t/L_a, C_o/K_a, S) \quad \text{Eq 7}$$

$$t = f\left(\frac{d^2, f_o^3, H_t, S, L_b}{Q^{1.5}, C_o, K_b}\right) \quad \text{Eq 8}$$

Where:

$C_o$  = influent concentration

$K_a$  = breakthrough index, minimum value

$K_b$  = floc strength index, maximum value

$d$  = grain size

$H_t$  = terminal head-loss

$L_a$  = depth of floc penetration, weakest flocculation

$L_b$  = depth of floc penetration, strongest flocculation

$S$  = surge amplitude

These equations showed how the design variables were related to the two principal design objectives: water quality and length of filter run. He suggested that equation 7 should be evaluated during the time of weakest flocculation and equation 8 should be evaluated in the time of strongest flocculation.

Eliassen (1941) was probably the first investigator to measure the rate of removal of turbidity at various depths of the filter throughout the run. The time rate of increase of head-loss was also studied.

The results indicated that the factors determining the time-rate of removal of suspended particles were as follows:

- a. Size of sand grains.
- b. Shape of sand grains.
- c. Variation of sand size with depth.
- d. Character of suspended particles in the water.

- e. Concentration of flocculated matter in the water.
- f. Amount of deposited particles already in the sand layer at each depth.
- g. Rate of filtration.
- h. Temperature of water.
- i. Porosity of the filter.
- j. Consolidation of the layer during the filter run.

Knowledge of each variable would be essential for the development of a complete theory of filtration; however, Eliassen did not formulate his results into a mathematical expression. His work has been frequently referred to by other investigators in many countries.

Significant results were obtained as shown in Figure 2 (p. 34). These curves showed that the removal was a maximum in the top portion of the unit for a considerable part of the run. As the upper pores became clogged, the section of maximum removal moved downward in the filter. He also observed that some iron was in the effluent at all times during the run. This indicated that the entire filter unit shared the burden of removal of suspended particles from the water, although the upper portion performed most of the removal at the beginning of the run. This observation of full penetration clarifies the common question about how deep the suspended particles penetrate in the filter.

Eliassen observed that the loss of head was directly proportional to time. However, he noted that deviation from



the straight line relation may occur irrespective of the length of time necessary to complete the run. The removal of solid matter from the water in the upper two inches of filter gradually became very small, approaching zero toward the end of the run. The loss of head of this layer, however, continued to build up at a practically constant rate, regardless of this saturation of deposits. This indicated that suspended particles were still being removed from the water in passing through this layer but of such a slight amount that the amount was not measurable in the test. Consolidation of the bed might have contributed to the increase of head-loss, particularly when filter runs were unusually long.

Geyer and Machis (1949) studied the penetration of solids into a filter bed under various filtration rates and various filter grain sizes. The plot of data showed that the increase in head-loss through the filter bed was proportional to the second power of the volume of water filtered. They reported that for a given amount of coagulated matter of character similar to that used in their tests, a 48 to 80 mesh (0.334 to 0.225 mm) sand produced 3 times the quantity of water produced by a 100 to 200 mesh (0.153 to 0.074 mm) sand to a terminal head-loss of 25 ft of water. The turbidity of the effluent for both cases was less than 1 ppm. They pointed out the higher filtration rates, even up to 10 gpm/sq ft in some cases, are possible without any deterioration in effluent quality provided the proper grain size and depth of filter

media were selected.

Stanley (1955) was the first investigator to use radioactive tracers in filtration study. He found that none of the deposited floc was dislodged and washed deeper into the filter, as the run progressed. Therefore, he disproved the dislodgement concept as suggested by Stein and Mints. In his paper, Stanley defined the "penetration" of flocculated matter as that depth where the radioactivity level was equal to the background level of radioactivity. This did not exclude the possibility of small amounts of floc continuously passing through the filter. He suggested a "penetration index" which was defined as the penetration in cm caused by the passage into a filter of 1 mg of Fe per sq cm of filter area.

For the experimental condition of his study he concluded:

- a. The head-loss increase varied approximately directly or showed a slight tendency to exponential increase with total Fe per sq cm.
- b. The "penetration" varied with  $d^{2.46} Q^{1.56}$  and varied directly as the total load of suspended matter applied per unit area of filter.
- c. The "penetration index" increased with an increase in floc size, varied approximately linearly with the sand size and varied directly as the flow rate. It was increased by NaCl, Na<sub>2</sub>SO<sub>4</sub> and MgSO<sub>4</sub> in solution and also varied with pH. It was at a minimum at pH of about 7.0 which he assumed to be

close to the isoelectric point.

Ling (1955) made extensive studies on filtration through uniform sands with sizes of 0.322 mm, 0.383 mm, 0.458 mm and 0.544 mm, using a graded sand filter as the control unit in which the sand had an effective size of 0.5 mm and uniformity coefficient of 1.6. He observed that the upper portion of the bed removed most of the turbidity during the first part of the run. The removal burden was gradually transferred downward as the filtration continues. This agreed with Eliassen's observations; the typical plot of percentage of turbidity removal per inch of depth against depth was shown in Figure 3 (p. 36).

The concept of equivalent depth was developed which was defined as the depth of the uniform sand required to have turbidity removal and loss of head identical with the control filter under the same operating conditions. He observed that the turbidity in samples taken from the lower portion of the graded sand bed was higher than that from the same portion of the uniform sand bed with 0.383 mm sands after the same period of filtration. If the same turbidity in the effluent was to be maintained for both filters, a shallower sand bed was required in a uniform sand filter. The head-loss generally was not identical between the uniform and graded sand bed.

Ling reported that the rate of change in head-loss with time was approximately linear in both the uniform sand filter and the graded sand filter. In the uniform sand filter, the

length of the filter run varied directly as the 2.32 power of the sand size and inversely as 1.48 power of the filtration rate.

Conley and Pitman (1960b) studied various rates of filtration from 2 to 35 gpm/sq ft on sand-anthracite filters. They found that the use of polyelectrolyte coagulant aid in the filter influent resulted in equal effluent quality at all rates. They observed the distribution of head-loss through the filter bed and concluded that acceptable run length could be obtained by permitting deep penetration of material into the bed. They stated that the penetration depth could be controlled by the use of the coagulant aid.

Robeck et al. (1962) emphasized that coagulation and filtration are really one process and should not be studied separately. They considered that the floc age and strength were more important than the floc size or density in the removal by filters. In discussing the paper by Conley (1965), Robeck stated that the most practical method for supplementing the jar test to select the optimum coagulant dose was to observe the filtrability on a small-filter which incorporates a means to observe incremental head-loss through various layers in the filter bed.

Cleasby and Baumann (1962) were concerned primarily with the selection of optimum filtration rates for sand filters, and described the depth-removal relationship in a graded sand filter for hydrous ferric oxide floc. In Cleasby's work two

different types of suspension were investigated. One type was a hydrous ferric oxide floc, and the other was calcium carbonate from a lime-soda softening process. Results revealed that with particles having considerable structural strength, such as calcium carbonate, the head-loss development during a run was not linear, but approached an exponential shape. At higher rates of filtration the head-loss development became more linear, and the greatest production of filtrate resulted at the rate where the head-loss development first approached linearity. When the hydrous ferric oxide floc was filtered, the head-loss development was linear, and no optimum production rate was observed. An optimum rate tendency was attributed to a predominant surface removal at low filtration rates. Cleasby pointed out that the effluent turbidity was not always satisfactory at the higher filtration rates; therefore, the optimum rate from the standpoint of production might not be a feasible rate due to unacceptable filtrate quality. For example, when filtrating a more unstable, and more highly turbid influent water, the optimum production rate had not been reached even at 6 gpm/sq ft although the production increased steadily as the filtration rate increased. However, the effluent quality was not acceptable, even at 3 gpm/sq ft. Therefore, with this type of water, the selected filtration rate should be governed by effluent quality criterion, while for the other type of water the optimum production criterion should be applied.

#### D. Summary of Status of Filtration Studies

From the literature cited above it is evident that considerable work has been done which has led to better understanding of the sand filtration process and produced useful results for practical application. The present status of filtration has been in a transition from an art to science. However, the basic removal mechanisms have not been supported unequivocally by experimental or theoretical work. The mathematical treatment is still restricted by unrealistic hypothetical conditions. The same traditional design criteria with somewhat more flexible allowance in the rate of filtration continue to be applied to the filtration of a great variety of waters. It has been difficult for sanitary engineers to design the filter in a rational way without a practical method to predict the filter performance beforehand.

The modern trend of process design for water treatment leads to an integral consideration of filtration and pretreatment. This idea may achieve practical application provided that the filter performance can be predicted conveniently and satisfactorily under various conditions which may result from change of the pretreatment operation. The method presented in this study may be used as a stepping-stone to reach such a goal.

### III. THEORETICAL BACKGROUND OF THIS STUDY

The concept of the probabilistic nature of filtration through granular media has been grasped by the writer. It may be illustrated qualitatively from basic probability theory.

#### A. Bernoulli Trial and Random Walk

In probability theory, repeated independent trials are called Bernoulli trials if there are only two possible outcomes for each trial and their individual probabilities remain the same throughout the trials (Feller 1950, pp. 135-137). It is usual to denote arbitrarily one of the two outcomes as "success" and the other as "failure". If the simple event "success" is given probability  $p$ , and "failure" is given probability  $q$ , the probability resulting in either a "success" or a "failure" will be equal to unity, or  $p + q = 1$ . Since the probability of achieving one success in a single trial is  $p$ , the probability of occurrence of  $k$  successes in a series of  $n$  trials will be  $p^k$  and that of  $(n-k)$  failures will be  $(1-p)^{n-k}$  or  $q^{(n-k)}$ . In accordance with the multiplication of probabilities of independent events, the probability of occurrence of a particular sequence of  $k$  successes and  $(n-k)$  failures is  $p^k q^{(n-k)}$ . This is the probability of one sequence only. Since there are many possible sequences, the probability of any sequence of occurrence containing  $k$  successes and  $(n-k)$  failures is expressed as follows:

$$P(k; n, p) = \binom{n}{k} p^k q^{(n-k)} \quad \text{Eq 9}$$

Where:

$\binom{n}{k}$  is an expression of  $\frac{n!}{k!(n-k)!}$  for  $k = 0, 1, \dots, n$

Equation 9 is a typical binomial distribution as shown in Figure 1 for several arbitrarily selected values of  $p$  and  $n$ .

The Bernoulli trials can be represented graphically by a particle moving on a vertical axis and starting from the origin. When the trial results in a success, let the particle move one unit step downward; if a failure, let it move one unit step around its existing position, i.e. no net downward displacement. Such individual movements (i.e., successes or failures) occur unpredictably governed by the values of  $p$  and  $q$ . This process is the so called "random walk" (Feller 1950, pp. 311-337).

Let  $P(z; n, p)$  be the probability that the  $n^{\text{th}}$  step takes the particle to the position  $z$ . In this event,  $z$  steps among  $n$  steps are directed to downward movement and  $(n-z)$  steps result in no net downward displacement. Therefore,

$$P(z; n, p) = \binom{n}{z} p^z q^{(n-z)} \quad \text{Eq 10}$$

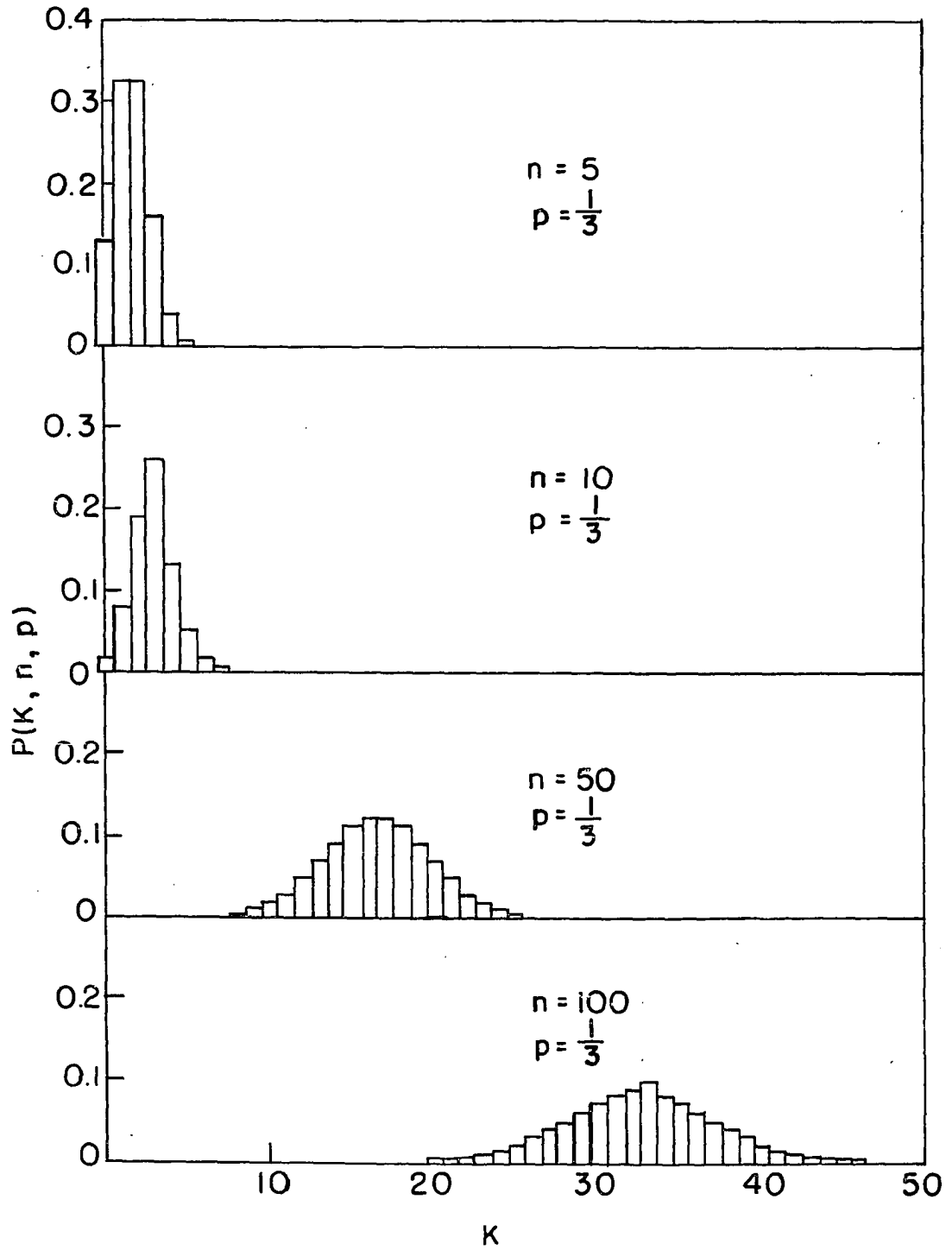
which is the same expression as equation 9.

#### B. Analogy between Random Walk and Filtration

In the case of filtration of suspended particles through granular media, the particles are carried by the flowing fluid which changes its direction randomly in an effort to find a relatively unobstructed pathway. The suspended particles,



Figure 1. Typical binomial distribution curves



even though they do not necessarily follow the same flow pattern as the fluid molecules, will also change randomly their directions as well as their speeds. Particles are brought by the resultant driving force, due to the interaction of several possible mechanisms, within the range of Van der Waal's forces of the granular filter medium or previously deposited particles, and will then adhere to surfaces exhibiting such forces. The point at which a suspended particle may adhere anywhere in the filter bed is therefore a matter of randomness.

It is noteworthy to cite Stein's findings on the observation of the removal of floc material within a visual filter bed which was made of parallel rods to simulate the sand. He found that a floc particle might contact a rod at a certain point; if it did not adhere at that point, it would be rolled along the surface to adhere at some other point on that rod or a lower rod, or to pass back into the flow. This observation supports the probabilistic nature of the clogging process which might be analogous to the model of random walk under the following conditions.

Consider a batch load of suspended particles in water of initial concentration  $C_0$  is entering the filter in an increment of time,  $\delta t$ , at a constant rate  $Q$  and make the following assumptions:

- a. The total number of particles  $\delta M$  of the influent in the increment of time,  $\delta t$ , is proportional to  $C_0 Q \delta t$ .

- b. The  $\delta M$  particles are identical in physical and chemical properties and spread uniformly over the filter area. Each particle performs random walk through filter bed without interference with the other particles. The probabilities  $p$  and  $q$  as defined before will remain constant and independent of the position of particle in the bed.
- c. Only the vertical component of the particle movement is of concern. The particle will perform the random walk at each unit step, starting from the top of filter bed.

In such a process, the total number of successive steps of random walk,  $n$ , will be proportional to  $rt$ , where  $r$  is the average rate of particle movement, and  $t$  is the time after the start of the walk. If the depth of filter to any point under consideration is  $L$ , the number of downward steps leading the particle to this point will be proportional to  $L$  and the probability of a particle moving to  $L$  at time  $t$  may be expressed as

$$P(L; rt, p) = \binom{rt}{L} p^L q^{(rt-L)} \quad \text{Eq 11}$$

The total number of particles arriving at  $L$  will be proportional to the total number of particles in the batch load which perform random walk,  $\delta M$ , in an increment of time,  $\delta t$ , multiplied by the probability of a single particle arriving at  $L$  within the same increment of time. Since the amount of suspended particles,  $\delta m$ , arrested in a lamina is

proportional to the amount that goes in as stated on page 8, then

$$\delta m \propto P(L; rt, p)\delta M$$

$$\text{or} \quad \delta m \propto P(L; rt, p)QC_0\delta t \quad \text{Eq 12}$$

By the continuity equation,

$$\frac{1}{Q} \frac{\partial \sigma}{\partial t} = - \frac{\partial C}{\partial L} \quad \text{Eq 2}$$

the decrease in the quantity of suspended particles is equal to the increase of deposited particles in the lamina. Within the increment of time,  $\delta t$ , the change of specific deposit,  $\delta \sigma$ , within this lamina due to the applied batch load will be equal to  $-\frac{\delta C}{\delta L} Q\delta t$ . Again the change of specific deposit,  $\delta \sigma$ , is proportional to  $\delta m$ , the total number of particles deposited within this lamina in an increment of time,  $\delta t$ . Therefore,

$$\frac{\delta C}{\delta L} Q\delta t \propto P(L; rt, p)QC_0\delta t \quad \text{Eq 13}$$

$$\text{or} \quad \frac{1}{C_0} \frac{\delta C}{\delta L} \propto P(L; rt, p) \quad \text{Eq 14}$$

Equation 14 shows that the turbidity removal per unit depth of filter bed could be expected to be governed by a probability law; under the foregoing hypothetical assumptions, the binomial law would apply. The actual conditions in filtration are different from the foregoing hypothetical assumptions. For example, the probabilities  $p$  and  $q$  in the random-walk model will no longer remain constant as the filtration progresses and will be different at different depths. Therefore, a simple binomial law can not describe the

actual process. However, the probabilistic nature is typical of rapid sand filtration as demonstrated by the experimental data from many workers which show this characteristic of probability distribution as shown in Figures 2 and 3 (Eliassen 1941, Ling 1955).

### C. Some Probability Distributions

The functional forms of some probability distributions are shown in Table 1. In these equations,  $X$  is a random variable in the probability distribution, usually named the variate. It is the same thing as  $k$  used in the binomial distribution; and  $U$  has been used in the chi-square distribution to distinguish it from the others. The shape of the probability distribution curve for any of these equations could be found by selecting any value of the variate,  $X$ , and calculating the value of the function,  $f(X)$ , using arbitrary values for the constants within the restrictions shown in Table 1 (Kenny 1951, Ostle 1964).

The binomial distribution can be approximated by a Poisson distribution when  $n$  is large and  $p$  is small; i.e. the larger the size of  $n$  and the smaller the size of  $p$ , the better the approximation. The binomial distribution can also be approximated by a normal distribution, the approximation is better the larger the size of  $n$  and the closer the value of  $p$  is to  $\frac{1}{2}$ , i.e.  $p = q$ . In case  $p$  is not equal to  $q$ , the binomial distribution is more skewed and is better approximated

Figure 2. Iron removal per unit depth vs depth  
at various filtration time (Eliassen  
1941)

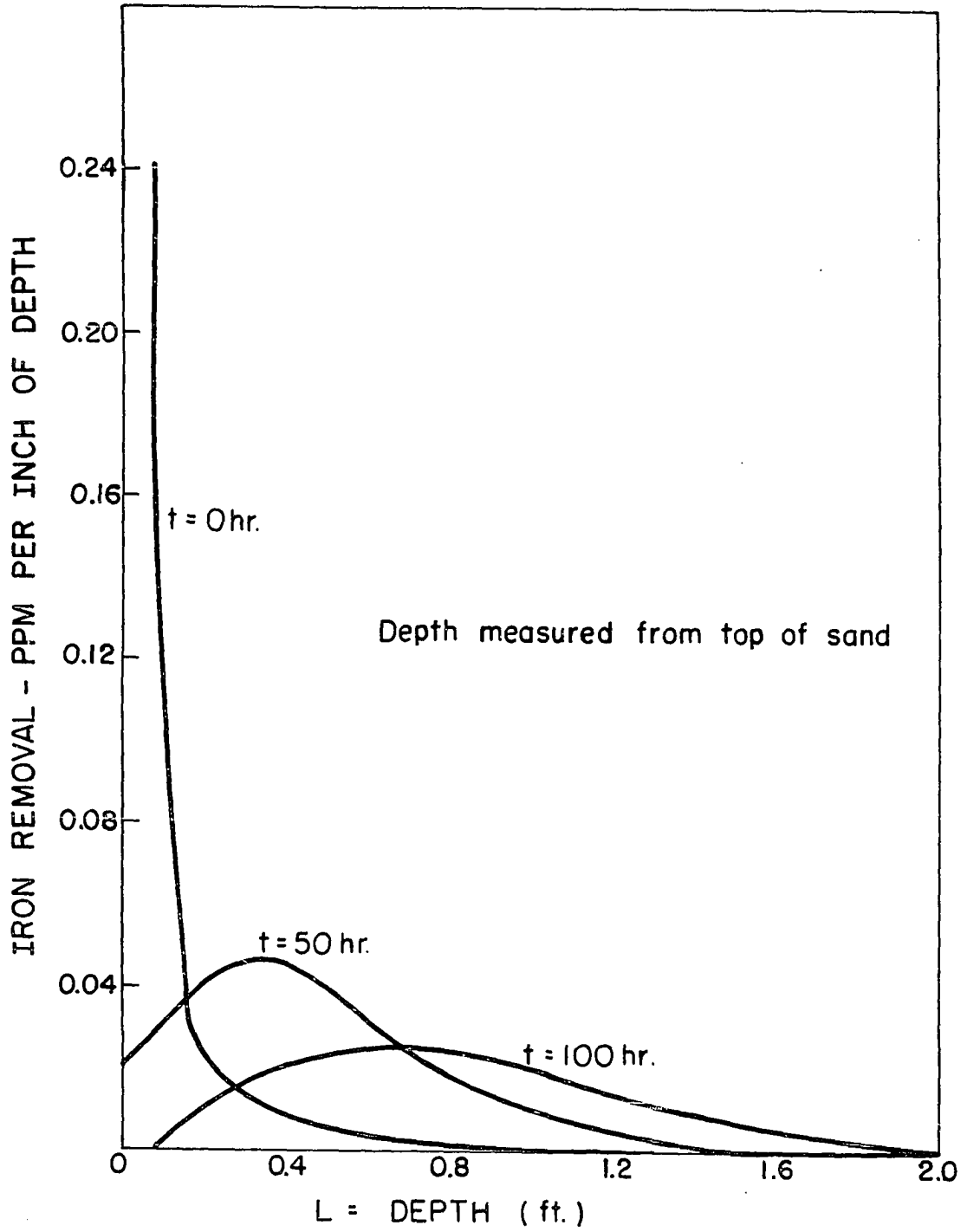




Figure 3. Turbidity removal per inch vs depth  
at various filtration time (Ling  
1955)

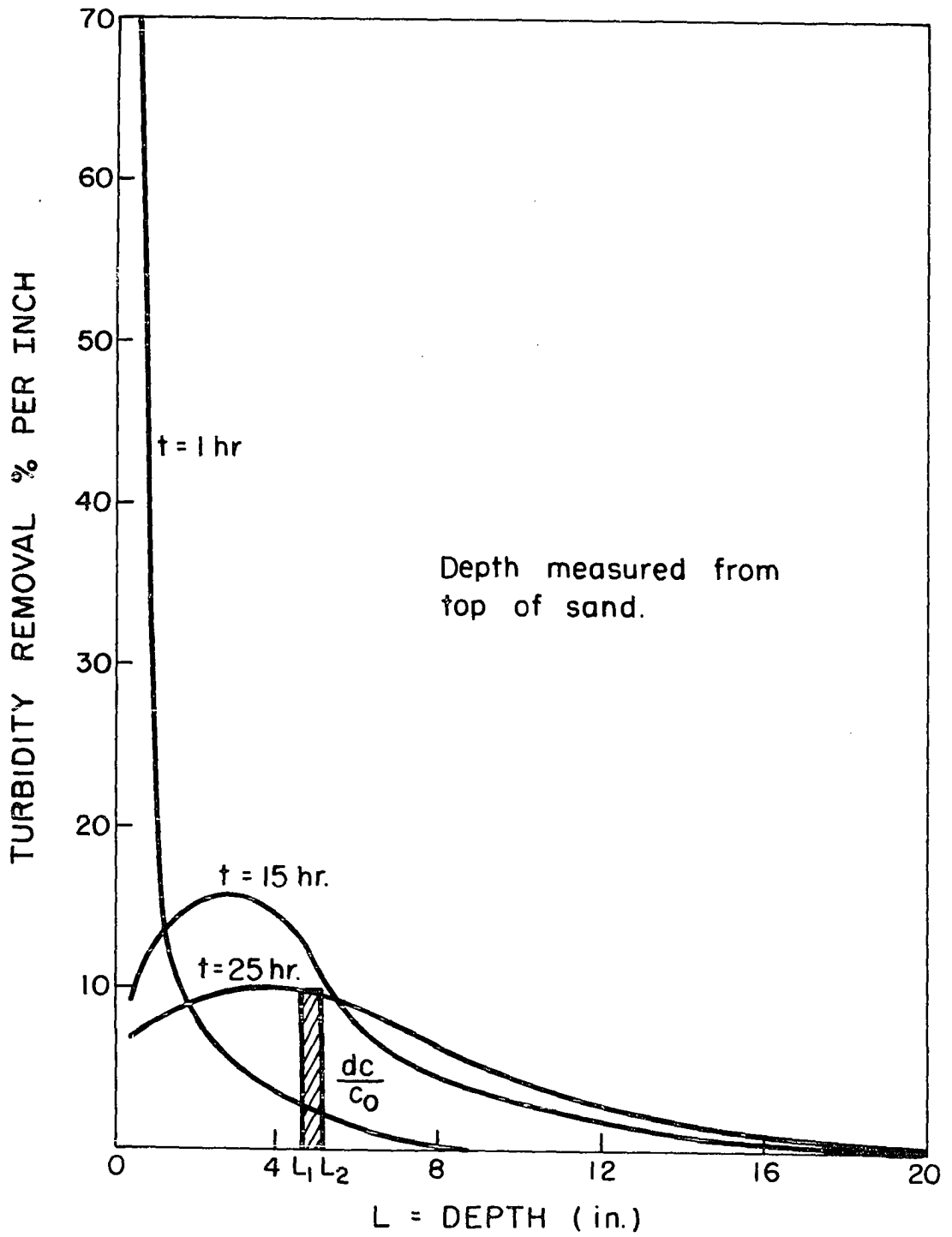


Table 1. Functional forms of some probability distributions

Name	Functional form*	Restrictions
Binomial	$f(k) = \binom{n}{k} p^k q^{n-k}$	$k=0, 1, \dots, n$ $0 < p < 1$ $n=1, 2, \dots$
Poisson	$f(X) = e^{-N} N^X / X!$	$N > 0$ $X=0, 1, \dots$
Normal	$f(X) = (2\pi C_1^2)^{-\frac{1}{2}} \exp[-(X-C_2)^2 / 2C_1^2]$	$C_1 > 0$ $-\infty < X < \infty$
Pearson Type III	$f(X) = C_3 X^{C_4} \exp(-X)$	$X > 0$ $C_3 > 0$ $C_4 > -1$
Gamma	$f(X) = \frac{X^{C_5} \exp(-X/C_5)}{\Gamma(C_5+1) C_6^{C_5+1}}$	$X > 0$ $C_5 > -1$ $C_6 > 0$
Chi-square	$f(U) = \frac{U^{(v/2)-1} \exp(-U/2)}{2^{v/2} \Gamma(v/2)}$	$U > 0$ $v=1, 2, \dots$

\*Where:

$$N = np$$

$C_1, C_2, C_3, C_4, C_5, C_6$  are constants.

$v$  is the degree of freedom in chi-square distribution.

$\Gamma$  is the symbol of gamma function, which is defined by  $\Gamma(v/2) = \int_0^\infty y^{v/2-1} e^{-y} dy$  as it appears in the chi-square distribution. If  $v/2$  is a positive integer,  $\Gamma(v/2) = (v/2-1)!$

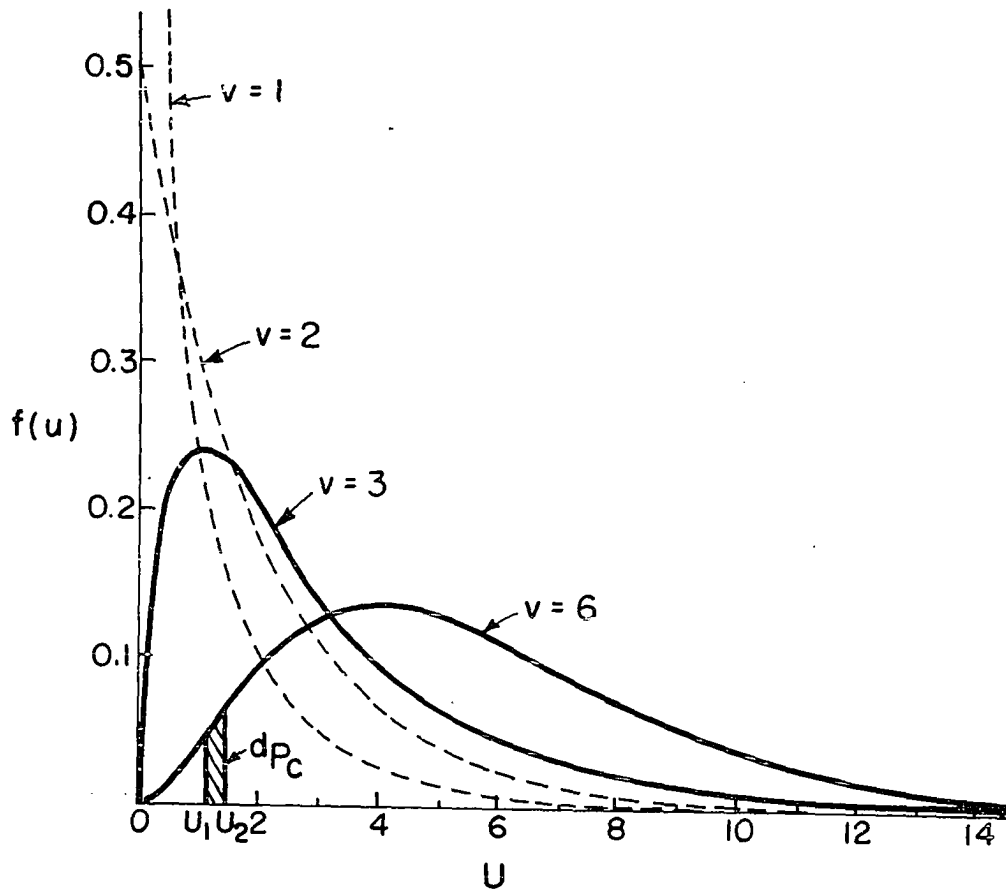
by a Pearson Type III curve than a normal distribution (Kenny 1951). It has been shown that the Gamma and chi-square distribution can be classified as Pearson Type III curves, and the chi-square distribution is a special case of the Gamma distribution. A typical plot for the chi-square distribution is shown in Figure 4. Karl Pearson further showed empirically that all of these probability distributions can be generalized to satisfy a single differential equation (Kenny 1951, pp. 94-105). Therefore, any one of these distributions may describe the filtration process to some extent.

In Figure 3, each curve represents a typical distribution for a specific  $t$ . The ordinate represents  $\frac{1}{C_0} \frac{\delta C}{\delta L}$ , the area under each curve between  $L_1$  and  $L_2$  is equal to  $(\frac{1}{C_0} \frac{\delta C}{\delta L}) \delta L$ , or  $\delta C/C_0$ . Considering small increments of  $C$  and  $L$ , the area under the distribution curve between  $L$  and  $L + dL$  is then equal to  $dC/C_0$ .

In Figure 4, for a given  $v$  the area under each curve between  $U_1$  and  $U_2$  is equal to  $f(U) \delta U$  or between  $U$  and  $U + dU$  is equal to  $f(U) dU$ . By definition, the cumulative probability,  $P_c = \int f(U) dU$  for a continuous random variable. The area under each curve between  $U$  and  $U + dU$  is then equal to  $dP_c$ .

It is worthy to note the similarity between Figures 3 and 4, which indicates a possibility of fitting the experimental data of filtration as shown in Figure 3 into probability distribution curves such as are shown in Figure 4. In order to do so, the functional relations between  $U$ ,  $v$ ,  $P_c$  and their

Figure 4. Typical chi-square distribution curves



respective counterparts  $L$ ,  $t$ ,  $C/C_0$  must be established. However, even if these relationships could be established, such a curve fitting would have to be done once for each filtration condition; therefore it would not be practical. Rather, a simple method which can furnish adequate information for design purposes is needed. The purpose of this study is to introduce the variate  $U$  of the chi-square distribution as a parameter for the prediction of filter performance by a simple graphical approach rather than trying to find the exact relationship between the filtration data and probability distribution parameters.

The chi-square distribution occurs so often in statistical problems that it merits special attention. The cumulative probability of the chi-square distribution for different degrees of freedom has been tabulated in many statistics books. A part of such a table is reproduced in Table 10 in the Appendix. In statistics, the degree of freedom,  $v$ , is restricted to positive integers. Therefore, two kinds of plotting, as shown in Figures 50 and 51 in the Appendix, have been prepared based on the tabulated values for convenience in interpolation of intermediate values.

## IV. OBJECTIVES AND SCOPE OF THIS STUDY

The objectives of this study are as follows:

- a. To evaluate the feasibility of introducing the variate from the chi-square distribution for the prediction of filter performance.
- b. To evaluate the effects of several variables that determine filter performance. These variables are:
  - size of filter media
  - depth of filter
  - rate of filtration
  - concentration of influent.

The temperature and the type of influent, and the material of filter media were kept constant.

- c. To establish a simple criteria by which the filter may be designed rationally with the information obtained by using thin layer filter units.

The experimental evaluation in this study covers only the filtration of hydrous ferric oxide suspensions through Ottawa sands. Available data which support the applicability of the proposed method to other waters and media will also be presented.



## V. PILOT PLANT APPARATUS

For the purpose of this study a pilot plant was constructed, consisting of a mix tank, a pump and two sets of filter apparatus. Apparatus A consisted of three thin layer filters of different depths with an influent rate control system. Apparatus B was a plexiglass filter, six inches in diameter with a rate controller on the effluent side.

The effluent quality of each thin layer filter in Apparatus A gives the filterate concentration for that respective depth. The information thus obtained is equivalent to that from a single deep filter with multiple sampling outlets at the same respective depths. This is true so long as all three thin layer filters are in the same condition using same influent concentration and flow rate. Expected advantages of this arrangement over a deep filter with multiple sampling outlets were:

- a. Disturbance of deposits within filter bed during sampling was avoided.
- b. There was no limitation on the rate of sampling, up to the total filtration rate, and therefore a large sample could be obtained easily and quickly.
- c. No deposits accumulated around the sampling devices to cause straining effects on the samples as might occur when multiple outlets were used.

Disadvantages of this system were possible uneven

distribution of influent among filters and variation of filter conditions in the three filters due to differences in back-washing of each filter. However, these were considered insignificant for practical purposes if the experiments were conducted with equal care for each unit.

All components of the pilot plant had a capacity sufficient to operate the four filters simultaneously at any rate up to 6 gpm/sq ft.

The main installations as shown in Figure 5 are described in the following paragraphs.

#### A. Mixing Tank and Pump

A mixing tank 3 ft high and 3.5 ft in diameter equipped with a slow speed paddle was used to provide the necessary detention time to complete the iron floc formation. Allowing 6 in. of freeboard, this 182 gal tank provided a 30 minute theoretical detention time at a rate of incoming tap water of 6 gpm. The tap water was controlled by a flow controller\* and was adjusted during each run to a constant temperature of  $25 \pm 0.5$  °C by blending the cold and hot water through an automatic blending valve.\*\*

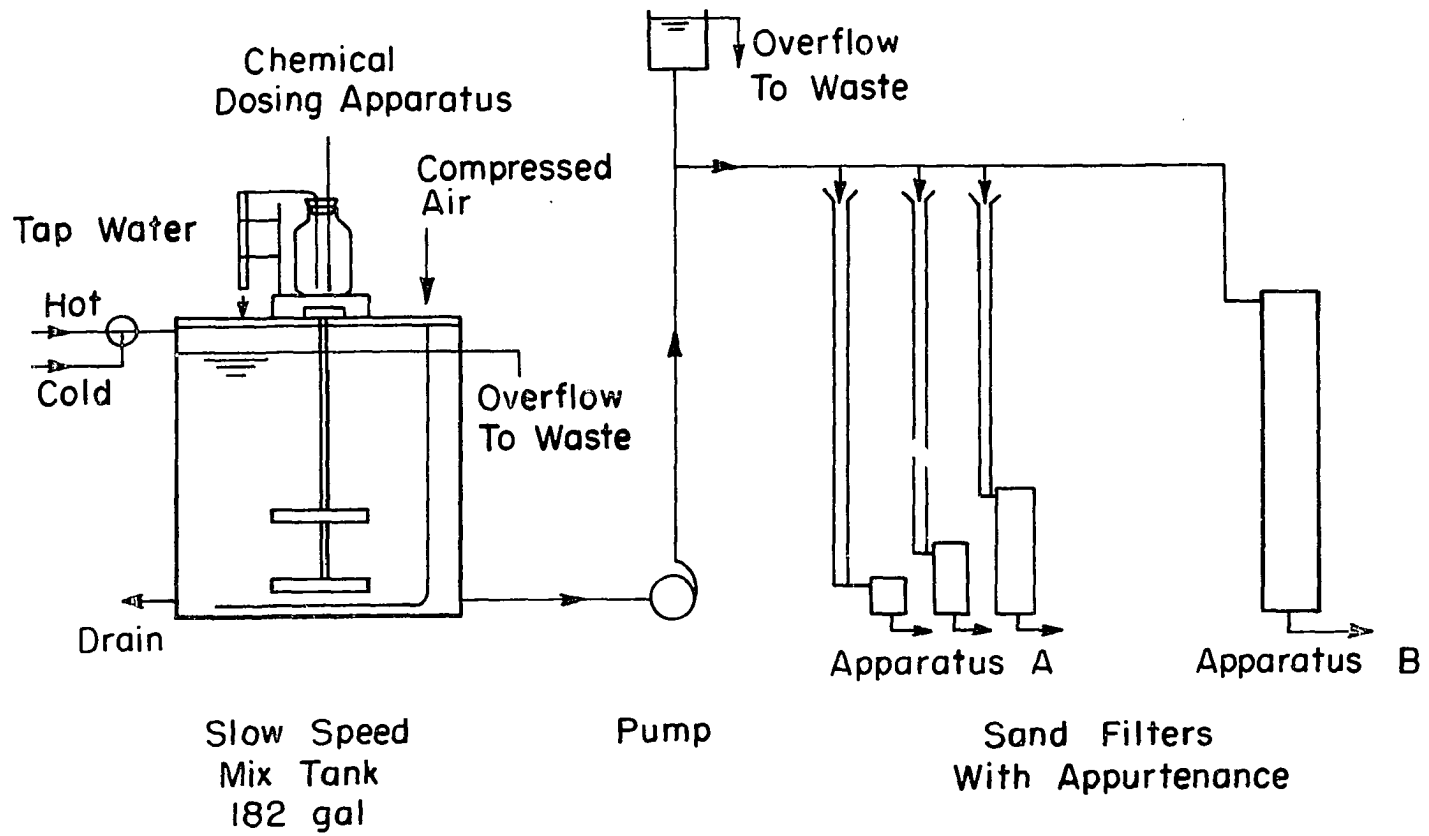
A 3/4 in. x 1/2 in., 1/3 hp, 3450 rpm close coupled centrifugal pump was used to pump water directly from the

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\* The Dole Valve Company, Morton Grove, Illinois.

\*\* Lawler Automatic Controls, Inc., Mt. Vernon, New York.

Figure 5. Schematic arrangement of pilot filters and auxiliary equipment



mixing tank to the filters.

## B. Filters and Appurtenances

### 1. Filter Apparatus A

Apparatus A consisted of three thin layer filters as shown in Figure 6. Each was constructed of a stainless steel shell, 4 in. in diameter which provided a filter area of 0.0873 sq ft. The three thin filter shells were 2 in., 6 in. and 10 in. deep respectively. The inlet pipe of each filter extended vertically to about 6 ft above the filter surface to receive the free fall pump discharge through a needle valve control. The inlet pipe was provided with a parallel pipe to release the entrapped air in the influent before entering the filter housing. A piezometer tube was connected to the filter top. The three pipes described above were transparent plexiglass.

The underdrain system consisted of one layer of #50 stainless steel mesh, strengthened underneath with a #10 mesh layer so that a flat bottom was obtained. A sampling tube of 1/4 in. diameter was connected to the effluent pipe immediately after leaving the filter and was allowed to flow continuously. The rest of flow passed through a flow meter and was disposed to the drain. At the beginning of each run, the influent flow control valve was adjusted to the desired rate as indicated by the flow meter before the sampling outlets were opened.

Figure 6. Filter Apparatus A and appurtenances

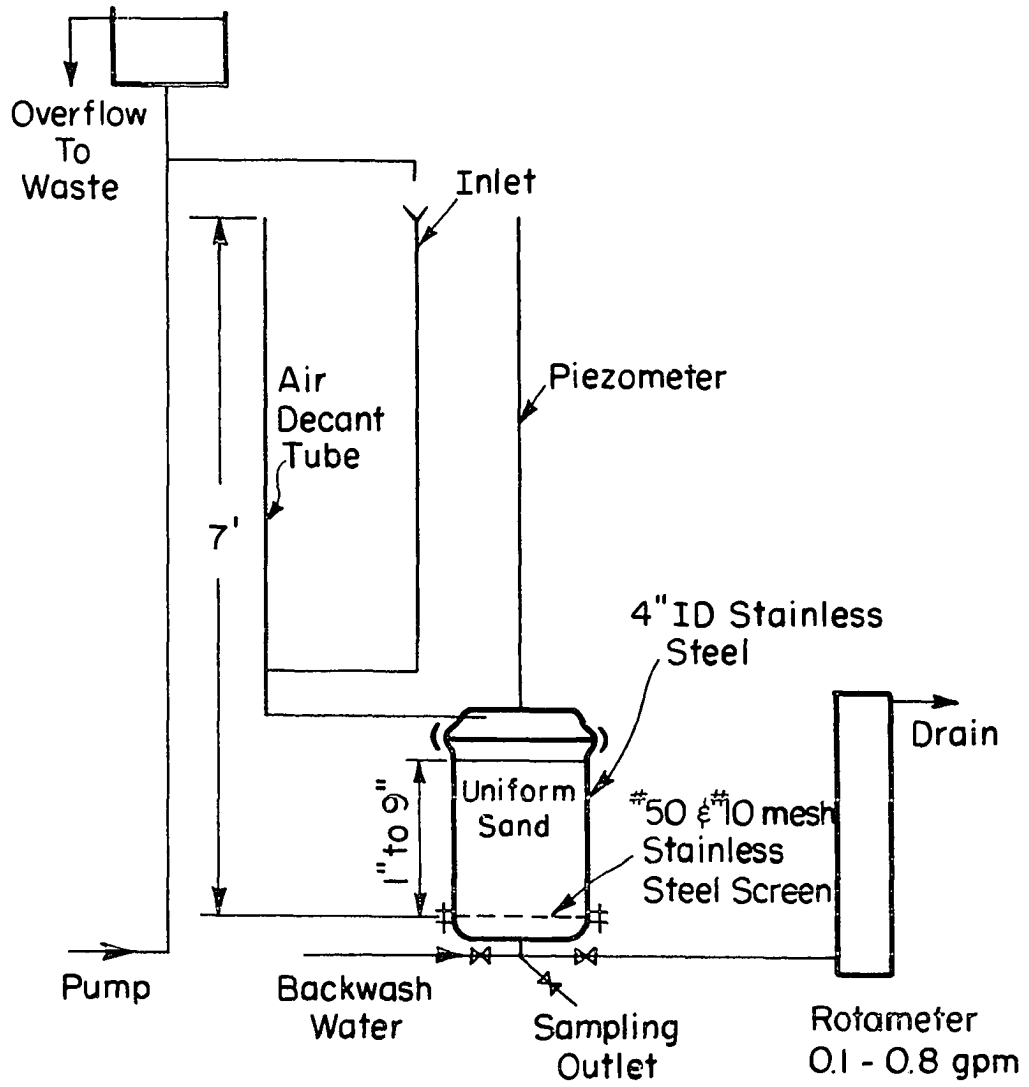
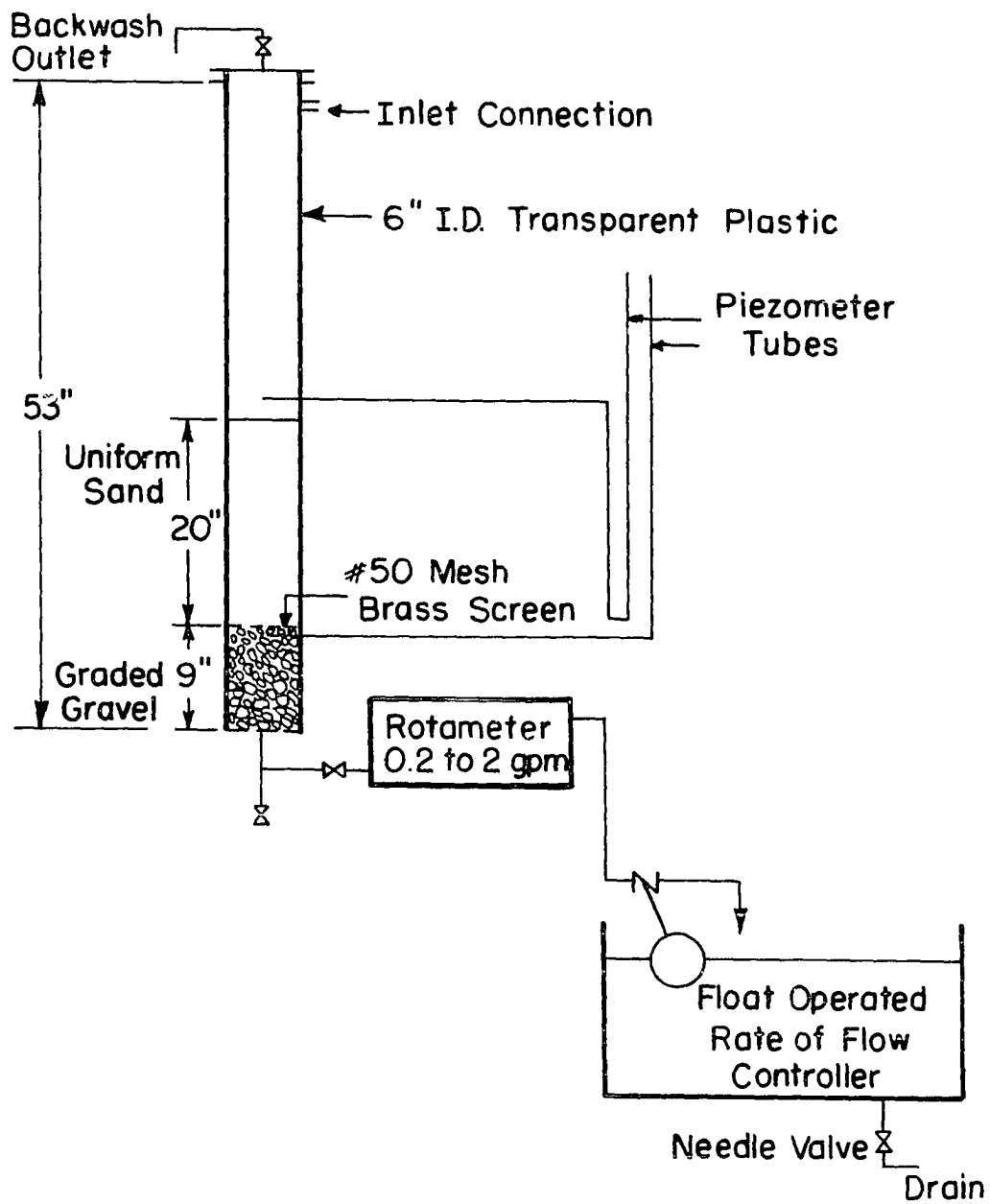


Figure 7. Filter Apparatus B and appurtenances





## 2. Filter Apparatus B

Filter Apparatus B was constructed of 6 in. I.D. plexi-glass tube 1/2 in. thick and 53 in. long as shown in Figure 7. It provided a sand surface area of 0.196 sq ft. The underdrain system consisted of 9 in. of graded gravel placed over a perforated aluminum cup which in turn was centered over the 1/2 in. filter effluent and backwash connection. On top of the gravel, a #50 mesh brass screen was placed to prevent the fine uniform sands from clogging the gravel or drain system. About twenty inches of uniform Ottawa sand was provided above the screen. Filter influent water entered approximately 23 in. above the sand surface. Piezometer tubes were connected to measure the loss of head across the sand bed.

## 3. Filter sand

The sand used throughout the runs was purchased from the Ottawa Silica Company, Ottawa, Illinois, which had an effective size of 0.435 mm and a uniformity coefficient of 1.38, a specific gravity of 2.65, and a porosity of about 40 per cent. The sand was then graded into several uniform sizes using U.S. Standard sieves. Samples of 500 g were sieved for a period of 10 minutes. The results are indicated in Table 2.

Uniform size, as defined for this study, means that 100 per cent of the sand passed a given U.S. Standard sieve and 100 per cent was retained by an adjacent sieve of larger number.

Mean grain size was referred in this study to the square root of the products of the sizes of openings of adjacent sieves. This is the procedure followed by a number of other filtration investigators.

Table 2. Filter sand sizes

Sieve No.	Size of opening (mm)	Mean grain size (mm)
20-25	0.841-0.707	0.771
25-30	0.707-0.595	0.649
30-35	0.595-0.500	0.545
35-40	0.500-0.420	0.458
40-45	0.420-0.354	0.386

#### 4. Flow meters

Effluent from each filter passed through a variable area glass, float type, flow meter suitable for water flow measurement from 0.1 to 0.8 gpm for Apparatus A, and from 0.2 to 2.0 gpm for Apparatus B. The flow meters were calibrated initially by collecting a timed sample in a two liter volumetric flask. Because of the deposition of small amount of iron on the inside of the glass tube and the float, the flow meters were cleaned periodically, about every five filter runs. At the same time, the filter sands were also cleaned with the same dilute hydrochloric acid solution with a pH value between

3 and 4. The acid solution was added to the flow meter tube and to the sand and allowed to soak over night and was then flushed out with clean tap water. During the investigation period, the flow meters were dismantled twice for more complete cleaning.

5. Rate of flow controller

The effluent from Apparatus B passed into a float operated rate of flow controller. The controller maintained a constant rate of filtration by holding a constant head on a needle valve outlet. As the head-loss through the filter increased during a run, the float valve would gradually open to maintain a constant filtration rate.

## VI. EXPERIMENTAL PROCEDURE

## A. Preparation of Influent Suspension

The filter influent water in all runs was prepared by continuously adding ferrous sulfate solution to aerated university tap water. The detailed procedure was as follows:

The tank was cleaned frequently to get rid of loose rust scales. Before each run, the tank was filled with university tap water, mixed for about thirty minutes and drained. The water temperature was adjusted to  $25 \pm 0.5$  °C by blending the hot and cold tap water. The filling of the tank was started simultaneously with feeding of the stock iron solution and the air. The bottom drain was kept slightly open to drain out continuously a small portion of the suspension to prevent accumulation of any heavy floc near the bottom. The mixer was started when the water level rose up to submerge the mixer paddles. About three hours after starting mixing, the concentration of the suspension became steady and filtration could commence. The stock iron solution, 0.1 M in concentration, was prepared by dissolving ferrous sulfate in distilled water containing 0.05 M hydrochloric acid to keep the iron in solution. A constant head capillary feeder was used to feed the ferrous sulfate solution to the water in the mix tank. The chemical feeding rate was checked periodically during each filter run to insure a constant rate of feed. When the room temperature fluctuated during a filter run, the viscosity of

the solution would change significantly, and the head was adjusted accordingly to maintain a constant rate of iron feed.

University tap water is a hard well water containing about 500 mg/l total dissolved solids, 320 mg/l of alkalinity and 450 mg/l of total hardness.

#### B. Measurement of Influent and Effluent Quality

Eliassen (1941) pointed out that one suspended component of a water is a positive index of the total suspended matter. Hudson (1962) cited examples of parallelism in removal of turbidity, manganese, microorganisms and bacteria and suggested that speed and simplicity make the turbidity measurement a valuable index of removal of other materials.

During preliminary investigations for this study a low angle turbidimeter was used for effluent quality monitoring. The turbidimeter was assembled according to the model developed by Black and Hannah (1965). However, a correlation between the micro-ampere reading of the turbidimeter with the iron content of the water and with the standard Jackson Turbidity Unit, could not be achieved. This was due in part to difficulties of controlling the scattering properties of iron floc and because of the deposition of iron within the turbidimeter sample cell and along the sampling lines. Therefore, during the remainder of the investigation, the effluent Quality was evaluated only by its iron content. The Colorimetric-Bipyridine method was selected for this study. In dilute

ferrous solution, with pH between 3.5 and 8.5, 2,2-bipyridine (dipyridyl) produces a pink color which obeys Beer's Law and is therefore suited to colorimetric determinations.

The normal method for the total iron determination involves dissolving and reducing any precipitated iron with acid and hydroxyl ammonium chloride before analysis. In some cases, heat is required to complete the solution of the precipitated iron. This procedure is very time consuming. Therefore, in this study, a patented bipyridine reagent "Biver" developed by the Hach Chemical Company, Ames, Iowa, was used to speed up the analyses. This reagent will dissolve and reduce the iron in a single step without heating. The developed color was observed on a Beckman Model B spectrophotometer, which was previously calibrated against standard iron solutions. The standard iron solutions were prepared by methods presented by Diehl (1960). The percentage of transmittance of light was observed at 512.5 m wave length.

The influent quality was measured using the same procedure.

### C. General Operating Procedure for a Filter Run

#### 1. Operation of filter Apparatus A

a. The same flow rate was set for each filter by adjusting the needle valves which controlled the filter influent. University tap water was filtered during this

period of valve adjustment which only took a few minutes.

b. The pump was then started and the valve on the discharge line was opened gradually until the flow rate to the filters reached the predetermined value, as indicated by the flow meters. Slight re-adjustment of the needle valves might be needed to achieve the desired rate for each filter. As originally designed, Apparatus A was equipped with a small constant head tank to maintain constant pressure on the needle valves. Early experience with this system was unsatisfactory because of iron deposits accumulating in the needle valves. Therefore, the above procedure was used thereafter.

c. The sampling outlets were then opened and kept open through the filter run to provide an adequate continuous sample.

d. The height of the water column in the piezometer indicated the over-all head-loss through the filter bed and piping system. The increase of head-loss across the sand bed within a time increment could be obtained from the difference of successive readings.

e. The first set of water samples for both influent and effluent were collected and initial head-loss readings were recorded within a few minutes after starting filtration. Subsequent head-loss readings and water samplings were made about every half or one hour. The length of filter run was about six hours. After the termination of the filter run the filter top was removed and an extension unit was attached for



backwashing. After about ten minutes backwashing, the backwash valve was closed slowly and the filter top was replaced with care to avoid any severe jarring action.

## 2. Operation of filter Apparatus B

a. The flow rate controller was set at a predetermined rate at the end of previous run.

b. Readings on two piezometers gave the head-loss across the sand bed. The head-loss readings and iron determinations were carried out in the same way as for filter Apparatus A.

c. The length of filter run was about ten to fifteen hours. After the termination of each filter run, the filter was backwashed for about ten minutes in the usual way. No extension unit was needed in the backwashing of filter Apparatus B.

### D. Miscellaneous Operating Procedures

Dilute hydrochloric acid solution at a pH value between 3 and 4 was used for cleaning the residual iron deposits on sand grains and in the flow meters which could not be washed out with tap water. Filter sands after being used for about five filter runs, were soaked over night in the acid solution and then backwashed with tap water.

After each backwashing a fairly consistent filter bed condition could be noted by the comparison of the initial head-loss reading, for different filter runs using the same flow rate and grain size.

## VII. SUMMARY OF EXPERIMENTAL RUNS

In order to evaluate the first objective of this study, a number of filter runs were conducted in several series as shown in Table 3.

In series A-1, uniform sands between #25 - #30 were used. All the sampling lines were connected to a common manifold connected to a sample cell in the low angle turbidimeter. Before taking samples from each filter, the system was thoroughly flushed with tap water. However, deposition of iron along the sampling lines and in the sample cell could not be avoided; particularly for the 1 in. deep filter, the measured iron contents were found to be lower than expected. Therefore, the data of this series has been excluded from further analysis.

Beginning with series A-2, the sampling lines were cut short and effluent samples were monitored by iron analyses as previously described. Samples were taken immediately after leaving the filters of Apparatus A. Effluent samples from Apparatus B were taken at the outlet of flow rate controller. Although the effluent line was comparatively longer in Apparatus B, deposition of iron was not likely because of the very low concentration of iron in the effluent. The same experiments were conducted in series A-1 and A-2 except for the sampling arrangement.

In series B, the sand size was changed to #30 - #35

and the same procedure was followed.

Series C-1 and C-2 were conducted, using three filters 1 in. deep to investigate the effects of sand size, flow rate and influent concentration on filter performance. However, experimental results in series C-1 using the sand between #20 and #25 were rejected because they gave an effluent concentration substantially higher than expected for a sand of this size (0.771 mm mean size). The probable reason was that the actual mean size was coarser than indicated. This might be due to the fact that the graded sand had a much lower fraction between #20 and #25. Therefore, when it was graded through a series of sieves, i.e. #20, #25, #30, #35, #40, #45 from top to bottom, the top two sieves would have a much lower load near the end of the sieving period than others underneath. Since the degree of separation was dependent of the residual load on any sieve, less fines would be expected in the coarser fraction. Thus, the different filtration effects observed might be attributed to different feeding loads during this series sieving technique.

Experimental runs in series D were conducted for comparison of the performance for uniform and graded sand filters.

Table 3. Summary of experimental runs

Series	Run No.	Filter bed conditions	
		Apparatus A	Apparatus B
A-1	#8 - #16	L = 1", 5", 9" d = 0.545 mm	L = 21" d = 0.545 mm
A-2	#17 - #25	L = 1", 5", 9" d = 0.545 mm	L = 21" d = 0.545 mm
B-1	#26 - #40	L = 1", 5", 9" d = 0.649 mm	L = 19.5" d = 0.649 mm
B-2	#60 - #63	L = 1", 5", 9" d = 0.649 mm	L = 18.3 " d = 0.649 mm
	#64	L = 1", d = 0.649 mm	-
C-1	#41 - #47	L = 1" D = 0.649, 0.771 mm	-
C-2	#48 - #51	L = 1" d = 0.545, 0.458, 0.386 mm	-
D	#52	L <sub>1</sub> = 1", d <sub>1</sub> = 0.458 mm L <sub>2</sub> = 1.45", d <sub>2</sub> *	-
D	#53, 54	L <sub>1</sub> = 1", d <sub>1</sub> = 0.458 mm L <sub>2</sub> = 1.25", d <sub>2</sub> *	-
D	#55	L <sub>1</sub> = 5.33", d <sub>1</sub> = 0.386 mm L <sub>2</sub> = 8", d <sub>2</sub> *	-
D	#56, 57	L <sub>1</sub> = 6.0", d <sub>1</sub> = 0.386 mm L <sub>2</sub> = 8", d <sub>2</sub> *	-
D	#58, 59	L <sub>1</sub> = 6", d <sub>1</sub> = 0.386 mm L <sub>2</sub> = 7.2", d <sub>2</sub> = 0.458 mm	-

\* Graded sand: effective size = 0.435 mm.  
uniformity coefficient = 1.38.

## VIII. FINDINGS AND DISCUSSIONS

## A. Relationship between U and Observed Filtration Data

1. Derivation of U values

The variate U from the chi-square distribution can be related to filtration data as follows:

a. Let  $C/C_0 = P_c$ , i.e., let the ratio of concentration at any time and depth to the influent concentration equal the cumulative probability in the chi-square distribution.

b. Let the filtration time, t, in hr = v, i.e. 1 hr = 1 degree of freedom.

c. Using observed values of  $C/C_0$  at various depths and times, U can be found readily by the use of Table 10 or Figure 50 or 51 in the Appendix, and plotted as in Figure 8.

d. When relating t to v, any unit may be used that is convenient. In this study all U values will be derived on the basis of the relation as mentioned in step b, otherwise, subscripts will be designated to distinguish the change. For example,  $U_{\frac{1}{2}}$  is used to represent that it is derived on the basis of 1/2 hr = 1 degree of freedom, and  $U_{100}$  designates that 100 hr = 1 degree of freedom as shown in Figures 9 and 46 respectively.

The observed experimental data and the derived U values for some typical runs have been presented in the Appendix, as examples of the results of the procedure outlined above.

## 2. Characteristics of the curves U vs t

When plotting the U value against t for each depth on log-log paper, the curves show a definite converging trend and approach a constant slope very similar to that as shown in Figure 51 in the Appendix.

The following results have been found interesting and very useful for developing the later part of this study:

a. The U value for a given filter depth increases significantly with t even if the filtrate quality does not change in a measurable amount. Such a definite trend is not evident when  $C/C_0$  is plotted against t for different depths.

b. The curves in such plot show not only a definite converging trend but also a typical shape regardless of the flow rate, grain size, and influent concentration being used. Figures 8 and 10 depict such characteristics for two typical runs, 22 and 40 in series A-2 and B-1 respectively.

c. The characteristic shape of the family of curves is not affected whether U, or  $U_{\frac{1}{2}}$  is used as shown in Figures 8 and 9; thus an arbitrary choice for the unit of t can be made in the method of prediction which will be presented later. For example  $U_{100}$  is used in Figure 48.

## 3. Equi-U curves using L and t as coordinates

For any arbitrarily selected U value, the corresponding L and t can be found from a U vs t plot as shown in Figure 10. These data can then be used as coordinates for plotting equi-U curves on log-log paper as shown in Figure 11. Straight lines

Figure 8. Run 22, U vs filtration time

$$d = 0.545 \text{ mm}$$

$$Q = 6.0 \text{ gpm/sq ft}$$

$$\bar{C}_c = 3.3 \text{ mg/l}$$

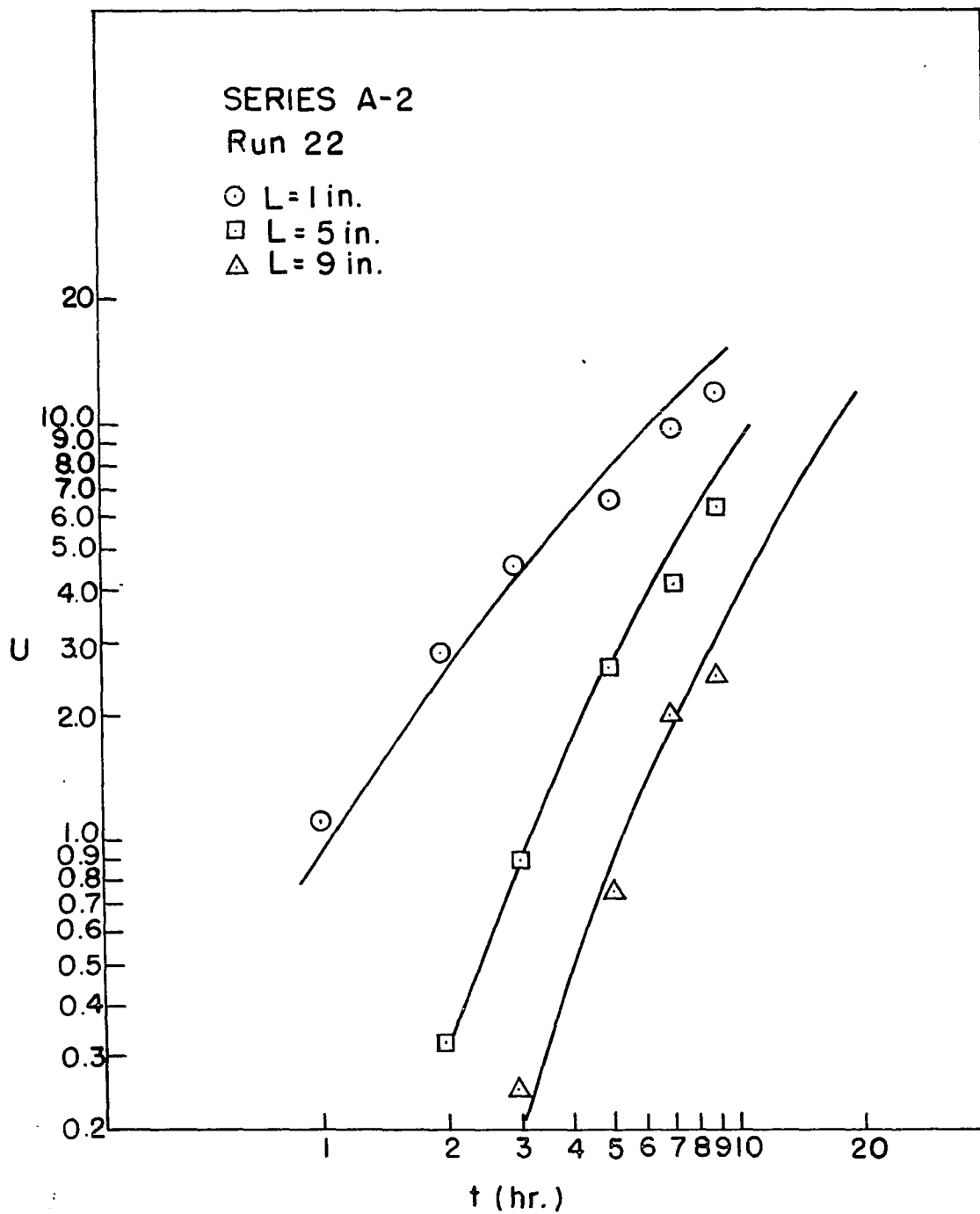




Figure 9. Run 22,  $U_{\frac{1}{2}}$  vs filtration time

$$d = 0.545 \text{ mm}$$

$$Q = 6.0 \text{ gpm/sq ft}$$

$$\bar{C}_0 = 3.3 \text{ mg/l}$$

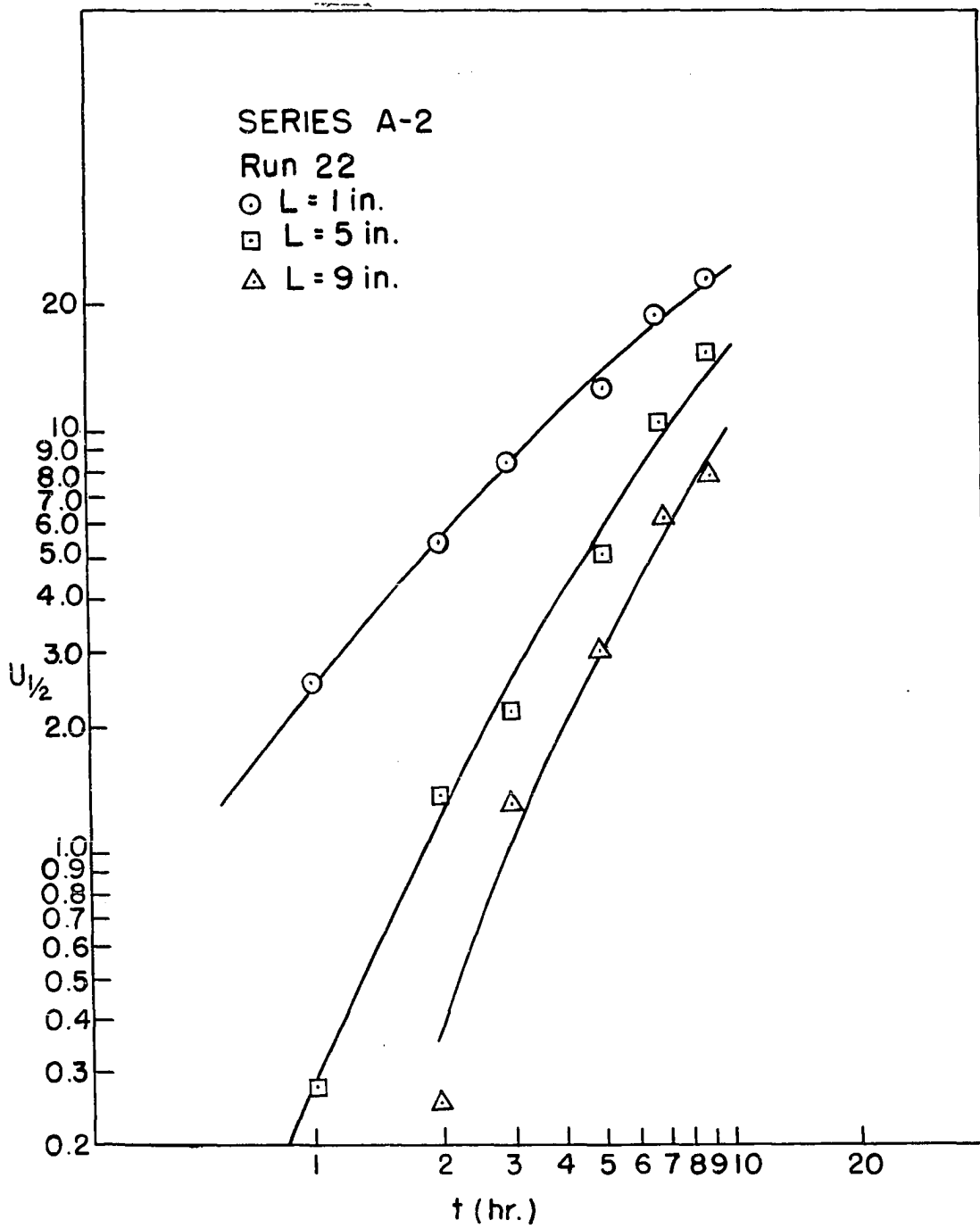
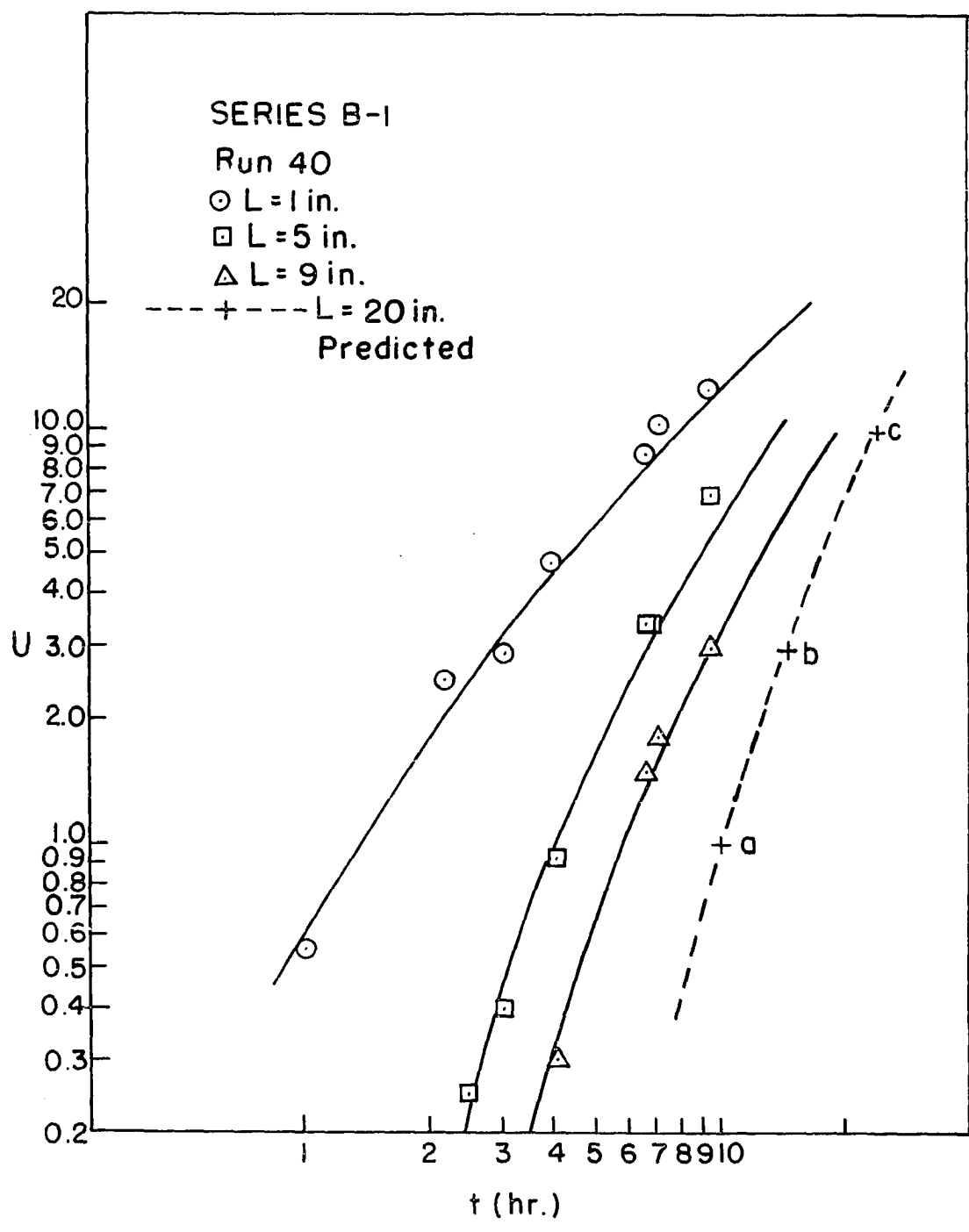


Figure 10. Run 40, U vs filtration time

$$d = 0.649 \text{ mm}$$

$$Q = 3.0 \text{ gpm/sq ft}$$

$$\bar{C}_o = 5.8 \text{ mg/l}$$



can be drawn by eye for good fit.

#### 4. Characteristics of the U vs $(H_t - H_0)$ curves

For any selected L and t, each corresponding U value can be plotted against the observed corresponding increase of head-loss,  $(H_t - H_0)$ , where  $H_t$  is the total head-loss up to the depth L under consideration, and  $H_0$  is the initial head-loss in a clean filter of the same depth L. Curves for three different depths are obtained as shown in Figures 12 and 13 which are similar in shape to Figures 8 and 10.

#### 5. Equi-U curves using L and $(H_t - H_0)$ as coordinates

For any arbitrarily selected U value, the corresponding L and  $(H_t - H_0)$  values obtained from Figure 13, can be used as coordinates for plotting equi-U curves on log-log paper as shown in Figure 14.

### B. Evaluation of the Effects of Filtration Variables Using U as a Parameter

The relationship of U to the filtration data has been described in the previous section which illustrates typical results for a given grain size, flow rate and influent concentration. It has been noted from the experimental results of series A-2 and B-1, that the curves such as shown in Figures 8 and 10, have the same shape and could be superimposed on each other by shifting either one along the abscissa, regardless of the differences in flow rate, grain size and influent concentration used in any particular filter run. In order to evaluate the individual effect of grain size,

Figure 11. Run 40, equi-U curves for L vs t

$$d = 0.649 \text{ mm}$$

$$Q = 3.0 \text{ gpm/sq ft}$$

$$\bar{C}_o = 5.8 \text{ mg/l}$$

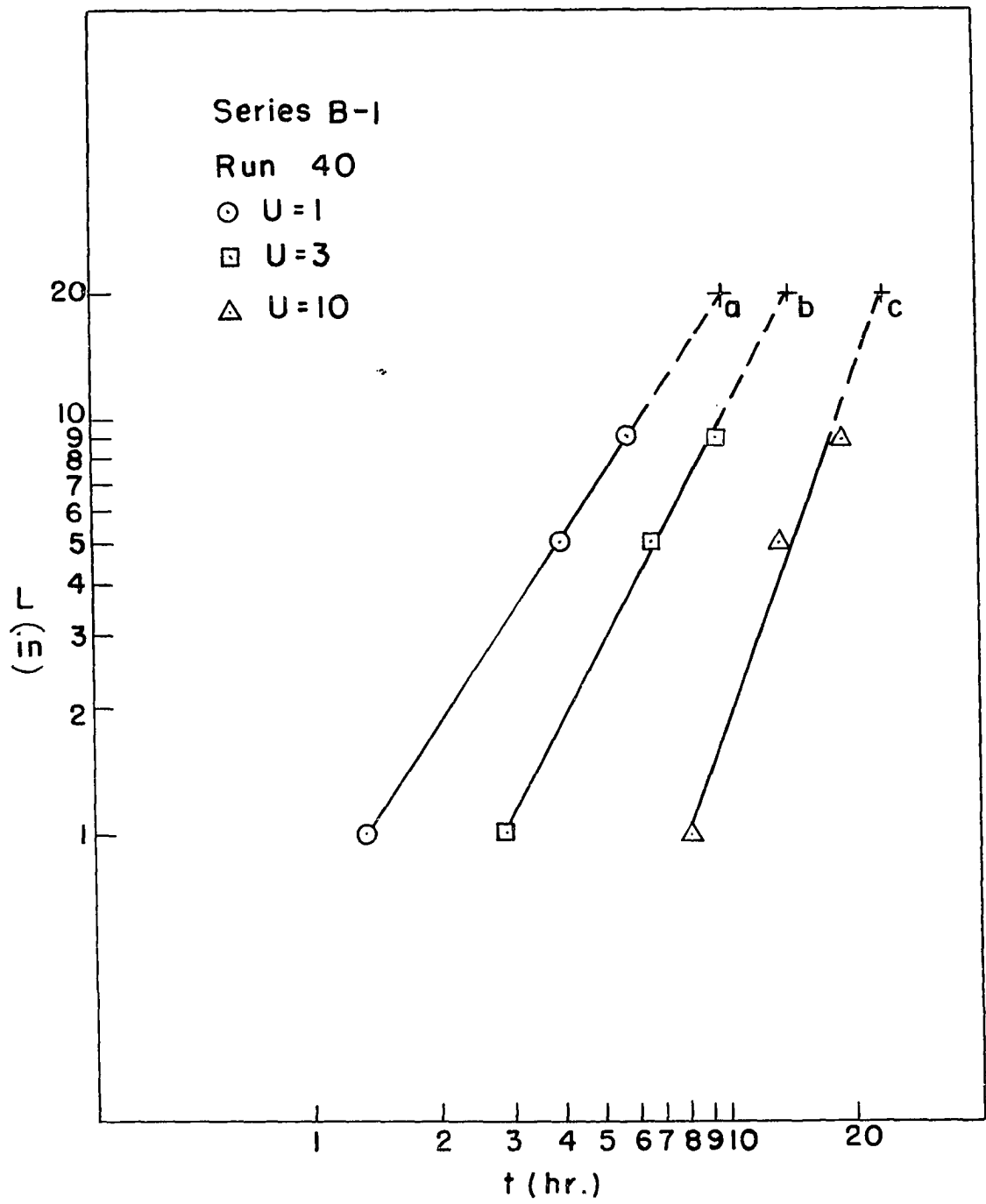


Figure 12. Run 22, U vs increase of head-loss

$$d = 0.545 \text{ mm}$$

$$Q = 6.0 \text{ gpm/sq ft}$$

$$\bar{C}_o = 3.3 \text{ mg/l}$$



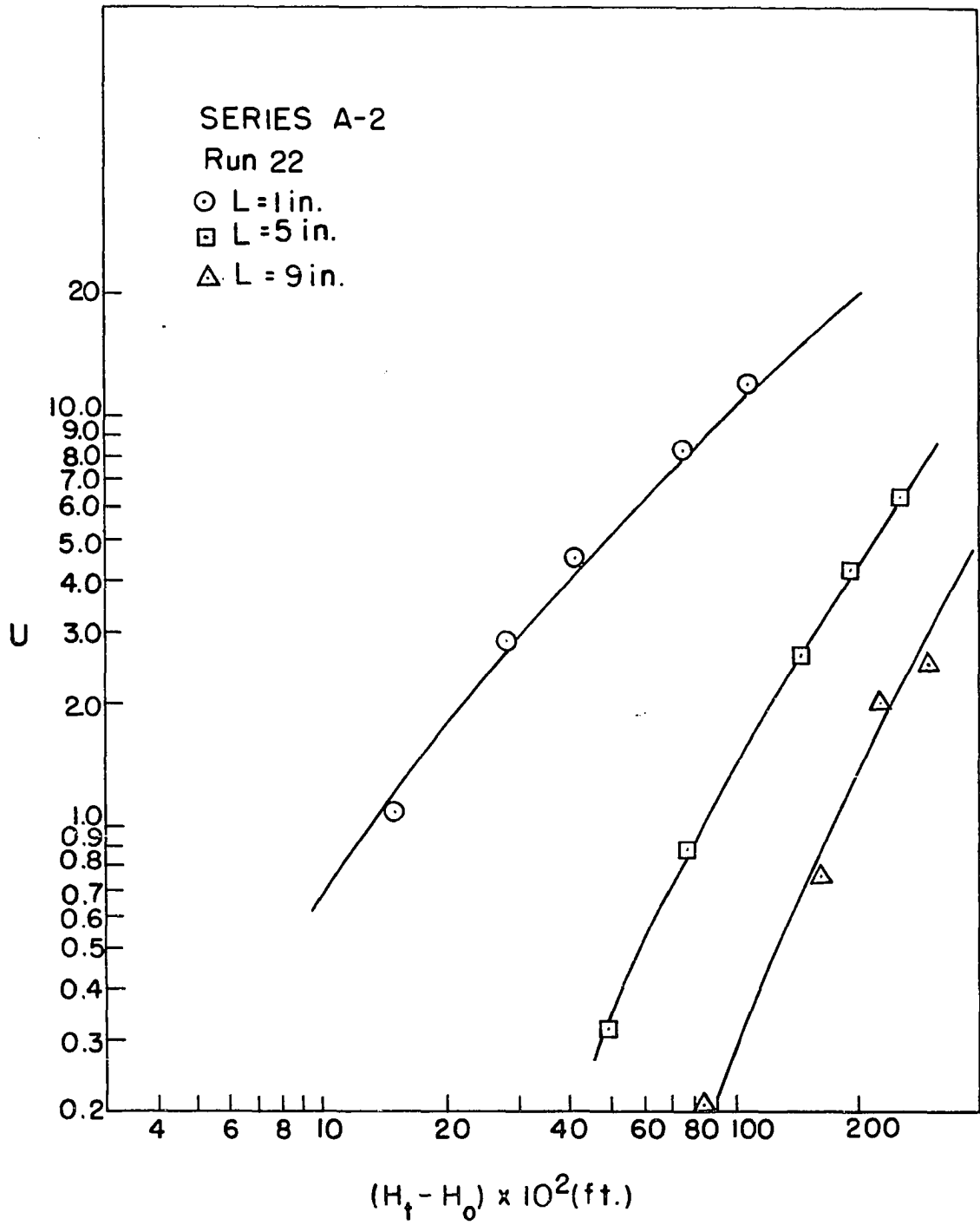


Figure 13. Run 40, U vs increase of head-loss

$$d = 0.649 \text{ mm}$$

$$Q = 3.0 \text{ gpm/sq ft}$$

$$\bar{C}_o = 5.8 \text{ mg/l}$$

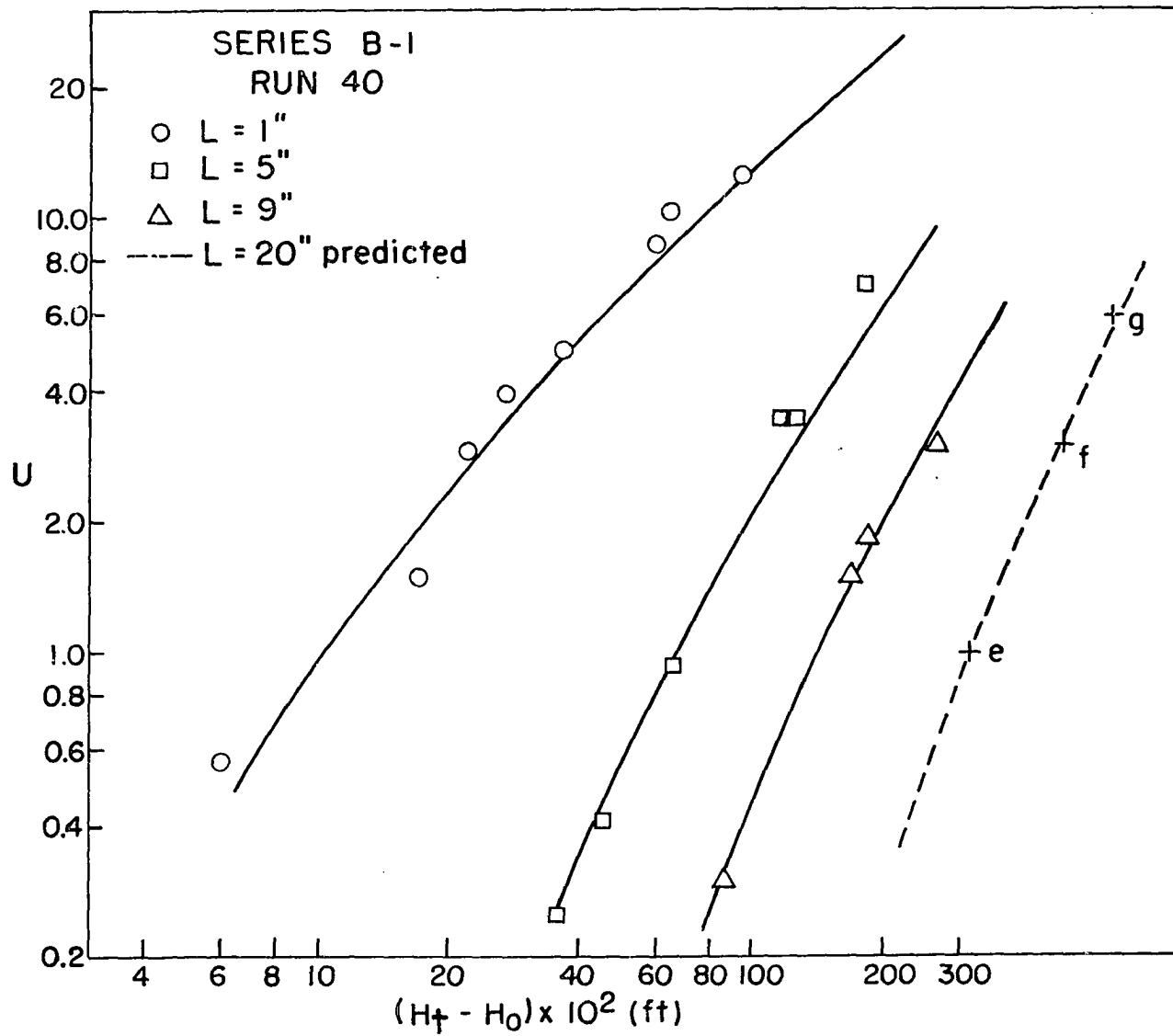
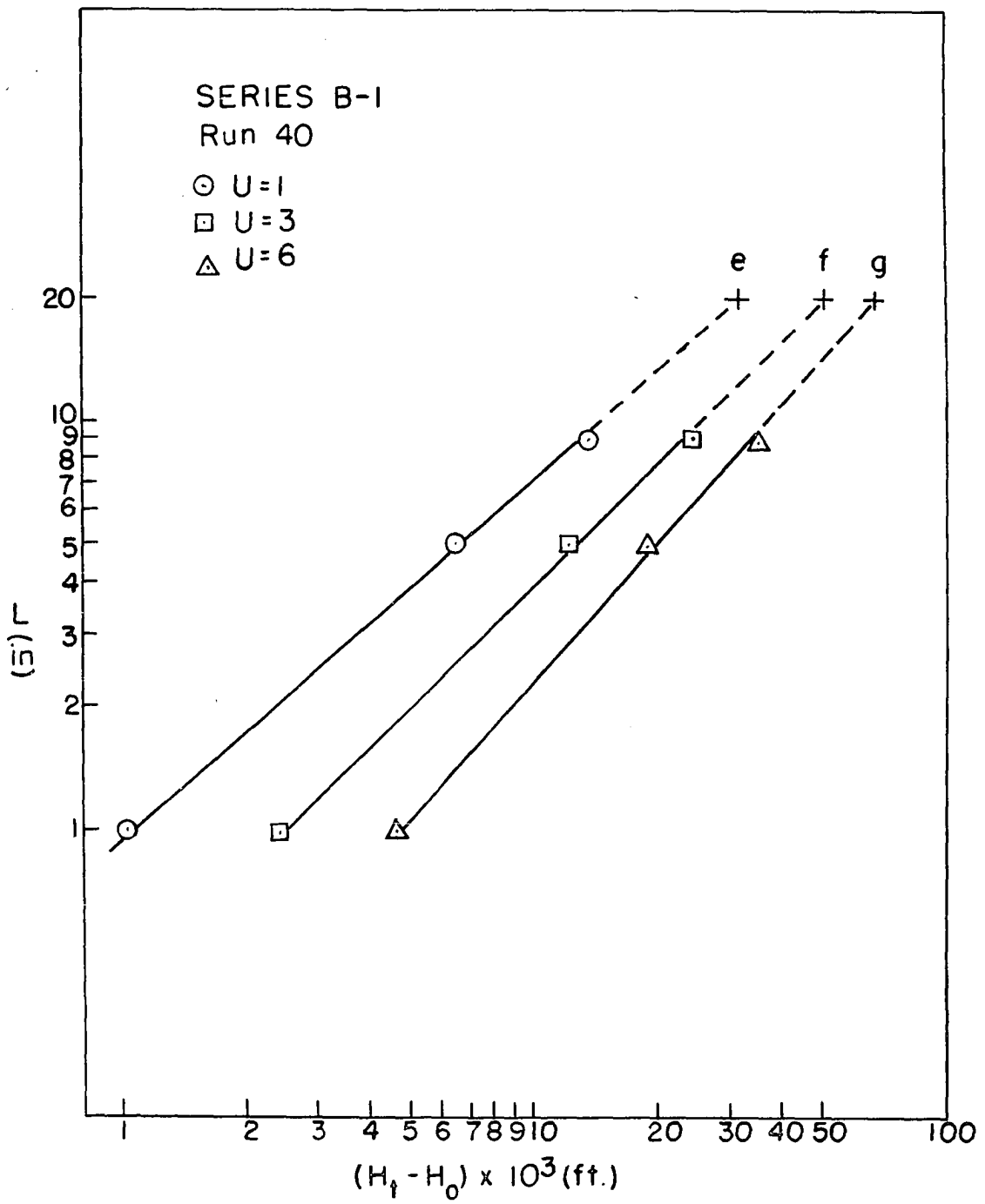


Figure 14. Run 40, equi-U curves for L vs  
increase of head-loss

$$d = 0.649 \text{ mm}$$

$$Q = 3.0 \text{ gpm/sq ft}$$

$$\bar{C}_o = 5.8 \text{ mg/l}$$



flow rate and influent concentration on the required shifting of such curves to achieve superposition, series C-1 and C-2 were conducted.

The effect of temperature on filtrability has not been investigated in this study.

#### 1. Effect of influent concentration on filtrate quality

Plots of  $U$  against  $t$  have been made for different filter runs in which the flow rates, grain sizes and filter depths were held the same but the influent concentrations were different, as shown in Figure 15. It is evident that the  $U$ - $t$  curves for runs 41, 43 and 45 coincide, indicating no measurable effect of the influent concentration on the percentage of removal within the range of these investigations. Therefore, it was apparent that when the influent concentration was increased, the effluent quality degraded even though the percentage of removal remained constant.

#### 2. Effect of flow rate on filtrate quality

Because there was no significant effect of influent concentration on the  $U$ - $t$  curve, the evaluation of other variables could be made regardless of variation of influent concentration between filter runs. The results of filtration runs using the same grain size and filter depth but different flow rates are presented in Figures 15 through 17. A  $equi-U$  plot for  $Q$  vs  $t$  on log-log paper is shown in Figure 18, with a slope approximately equal to  $-1/0.29$ ; therefore,  $\log Q \propto (-1/0.29)\log t$  or  $t \propto 1/Q^{0.29}$ . Furthermore, for a given value

Figure 15. Run 41, 43 and 45, U vs filtration time  
for different influent concentrations

$$d = 0.649 \text{ mm}$$

$$Q = 6.0 \text{ gpm/sq ft}$$

$$L = 1 \text{ in.}$$

$$\bar{C}_o = \begin{array}{l} 5.8 \text{ mg/l for Run 41} \\ 3.5 \text{ mg/l for Run 43} \\ 5.9 \text{ mg/l for Run 45} \end{array}$$

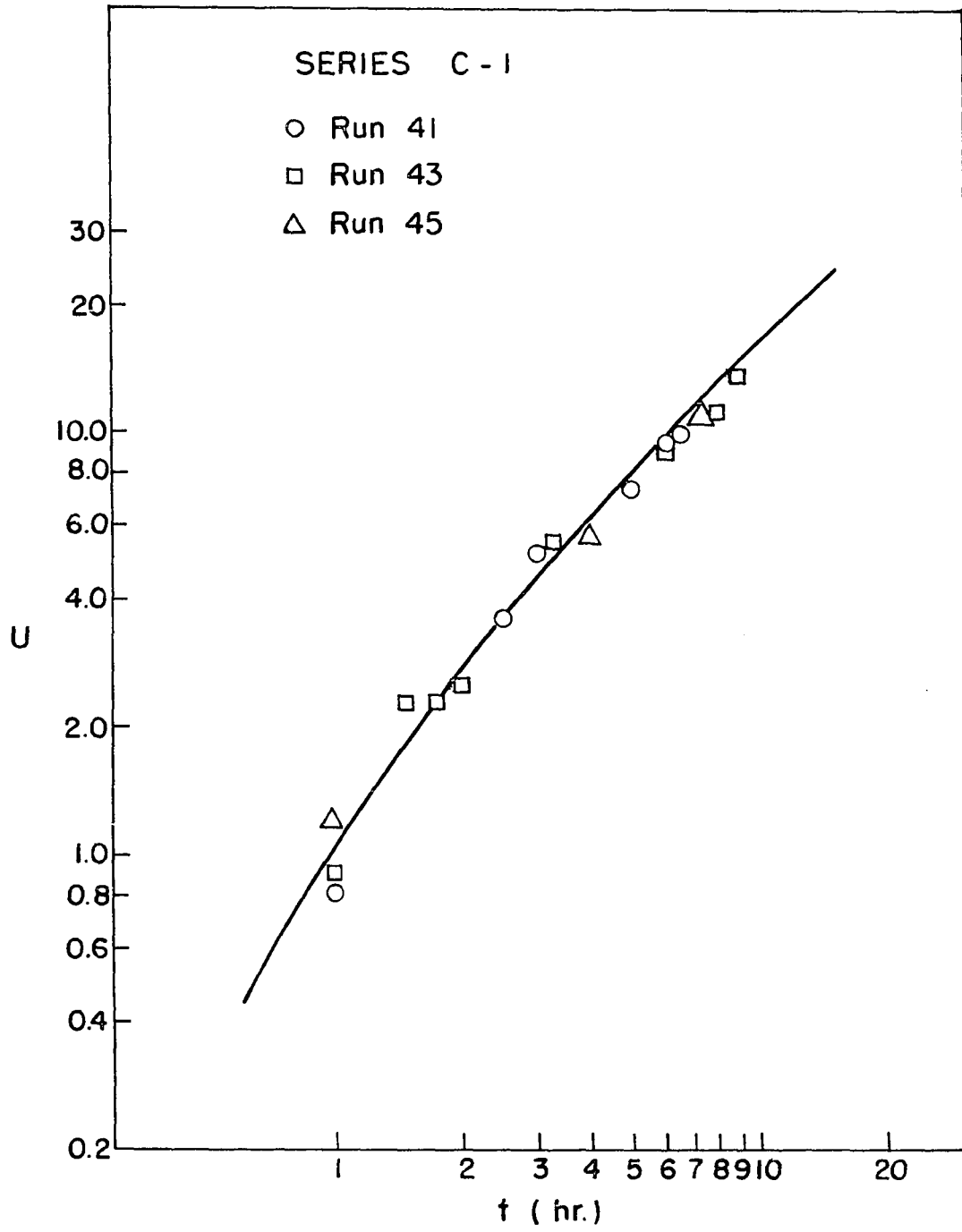




Figure 16. Run 44 and 47, U vs filtration time

$$d = 0.649 \text{ mm}$$

$$Q = 4.5 \text{ gpm/sq ft}$$

$$L = 1 \text{ in.}$$

$$\bar{C}_o = \begin{array}{l} 3.4 \text{ mg/l for Run 44} \\ 6.0 \text{ mg/l for Run 47} \end{array}$$

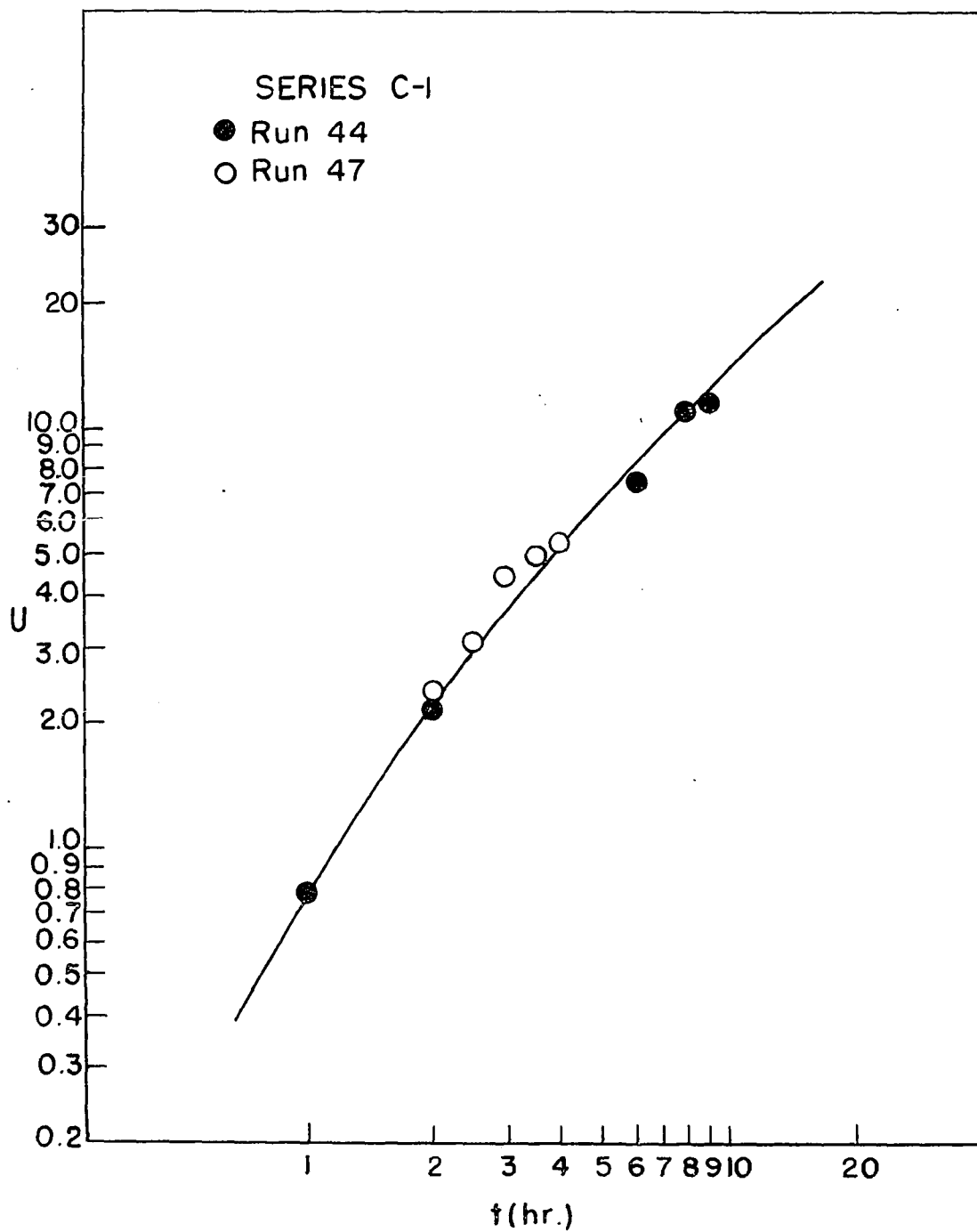


Figure 17. Run 42 and 46, U vs filtration time

$$d = 0.649 \text{ mm}$$

$$Q = 3.0 \text{ gpm/sq ft}$$

$$L = 1 \text{ in.}$$

$$\bar{C}_o = 3.4 \text{ mg/l for Run 42}$$

$$5.9 \text{ mg/l for Run 46}$$

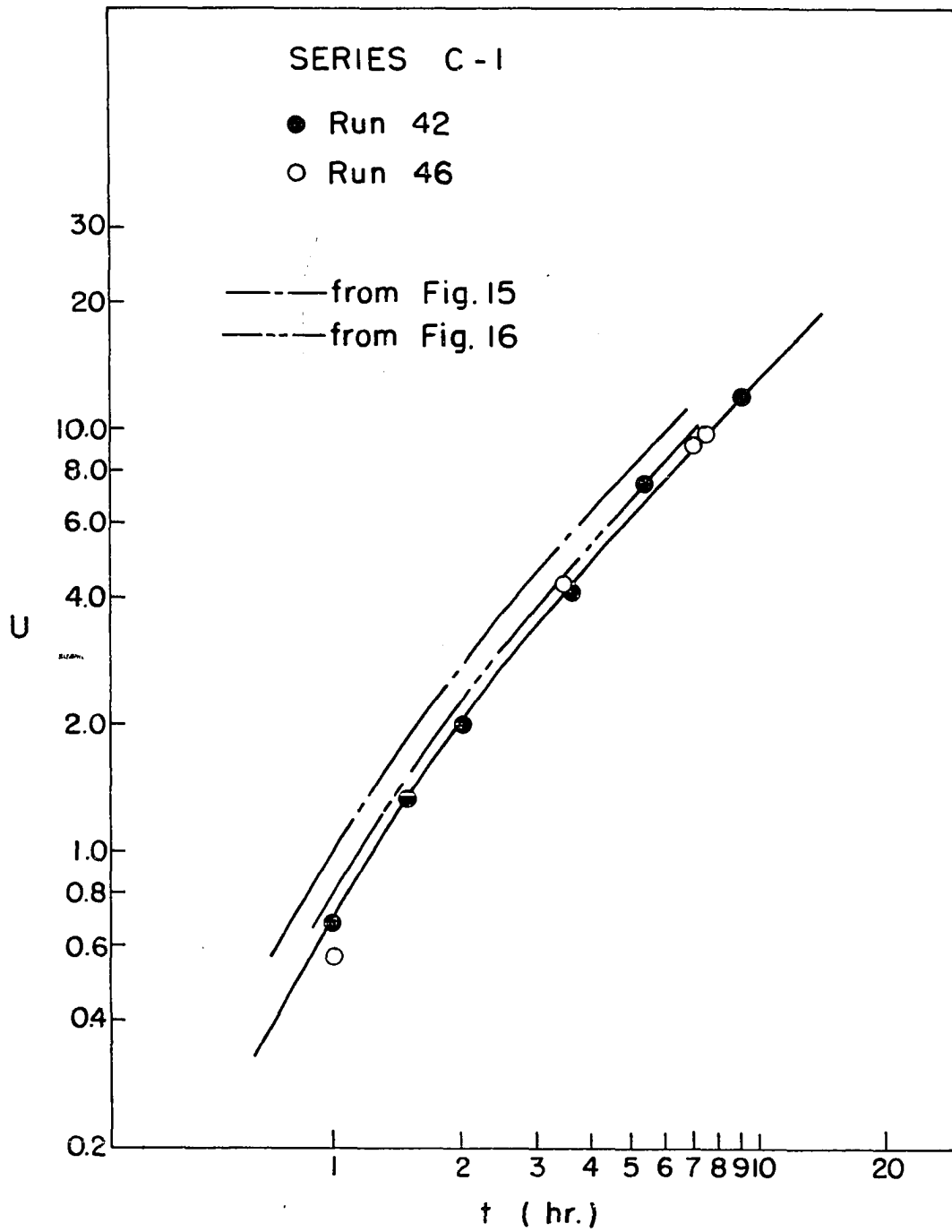
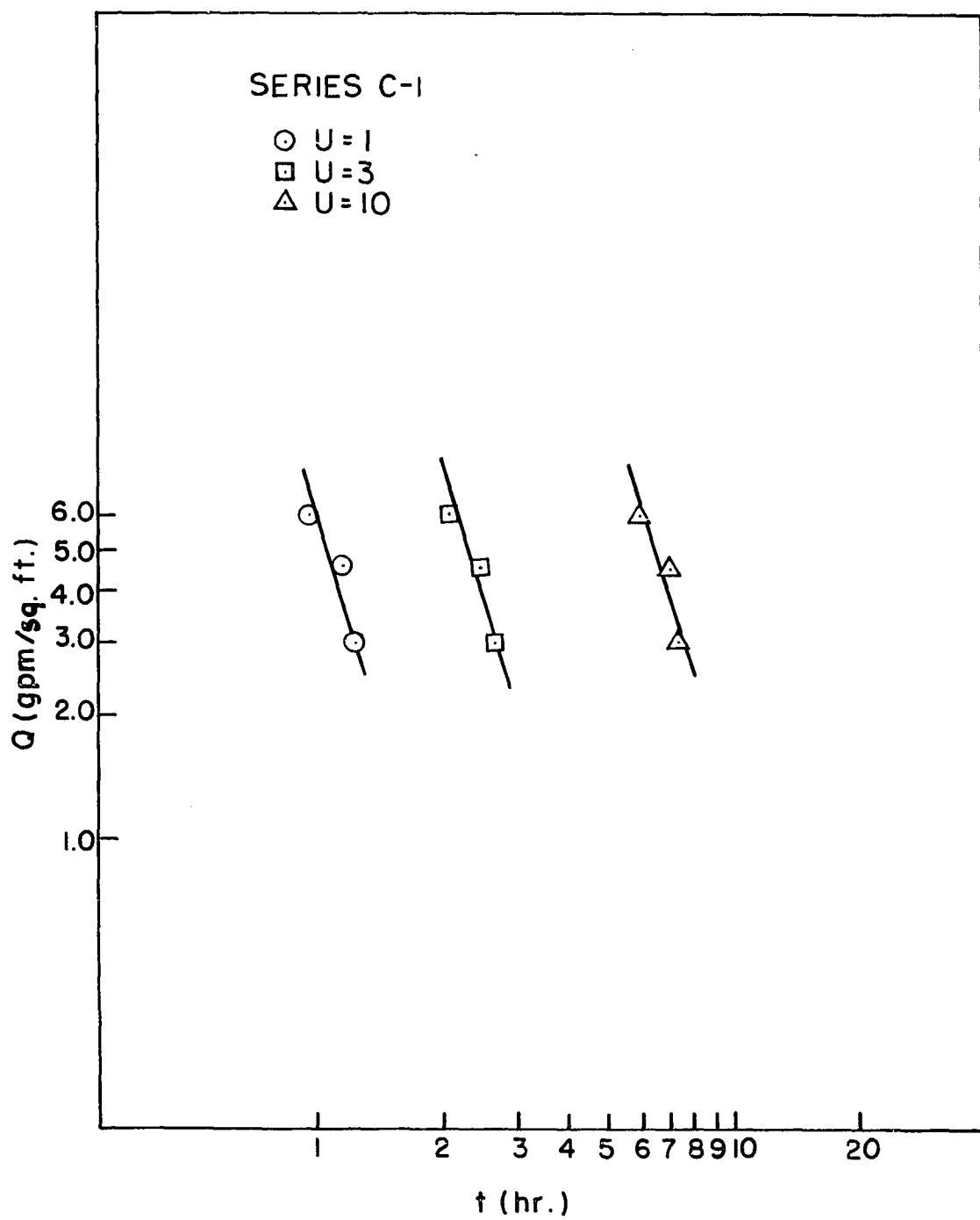


Figure 18. Relation of flow rate to filtration  
time for constant U

$$d = 0.649 \text{ mm}$$

$$L = 1 \text{ in.}$$



of  $Q^{0.29}t$ , the  $U$  value is constant for the depth  $L$  under consideration.

### 3. Effect of grain size on filtrate quality

Similar curves for the same flow rate and filter depth but different grain sizes are presented in Figure 19 through 22 which indicate the grain size effects. The slope of the equi- $U$  curves for  $d$  vs  $t$  on log-log paper as shown in Figure 23 is approximately equal to  $-1/0.62$ ; therefore  $\log d \propto (-1/0.62) \log t$  or  $t \propto 1/d^{0.62}$ , and for given value of  $d^{0.62}t$ , the  $U$  value is constant.

### 4. Effect of flow rate on head-loss

The same technique can be applied to evaluate the effect of these three filtration variables on head-loss. The results shown in Figure 24 indicate the effect of flow rate on head-loss. For a given  $U$  value the slope of the plot of  $Q$  vs  $(H_t - H_o)$  is approximately equal to  $1/1.2$  as shown in Figure 25; therefore  $\log Q \propto (1/1.2) \log (H_t - H_o)$  or  $(H_t - H_o) \propto Q^{1.2}$ , and for a given value of  $(H_t - H_o)/Q^{1.2}$ ,  $U$  is constant.

### 5. Effect of grain size on head-loss

Figure 26 shows the effect of grain size on head-loss. For a given  $U$  value, the slope of the curve of  $d$  vs  $(H_t - H_o)$  is approximately equal to  $-1/2.5$  as shown in Figure 27; therefore  $\log d \propto (-1/2.5) \log (H_t - H_o)$  or  $(H_t - H_o) \propto 1/d^{2.5}$ , and for given value of  $(H_t - H_o)d^{2.5}$ , the  $U$  value is constant for the given depth.

Figure 19. U vs filtration time for different grain sizes

$d = 0.649$  mm from Figure 15

$d = 0.545$  mm from Figure 20

$d = 0.458$  mm from Figure 21

$d = 0.386$  mm from Figure 22



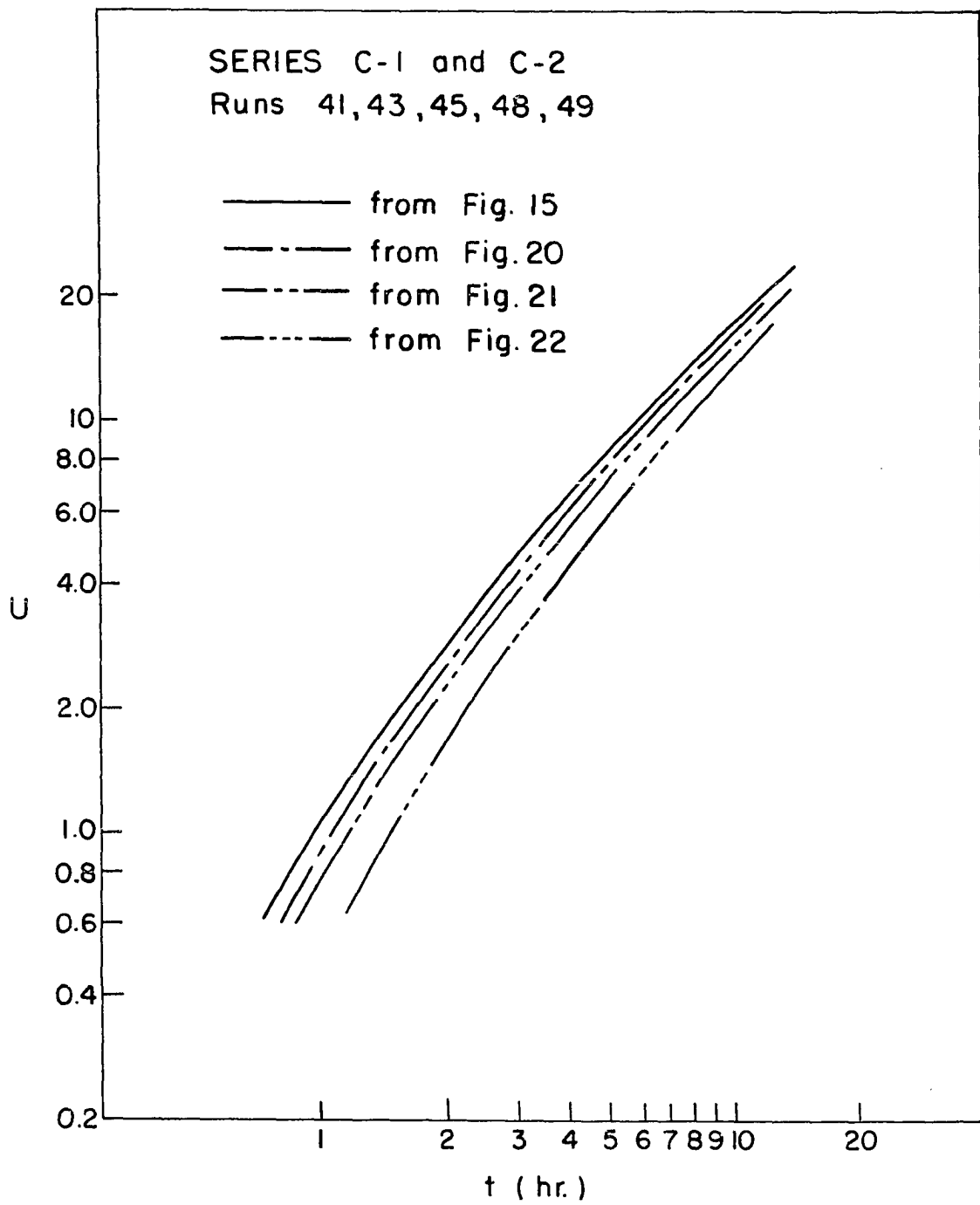


Figure 20.  $U$  vs filtration time for  $d = 0.545$  mm

$$Q = 6.0 \text{ gpm/sq ft}$$

$$\bar{C}_o = 3.4 \text{ mg/l}$$

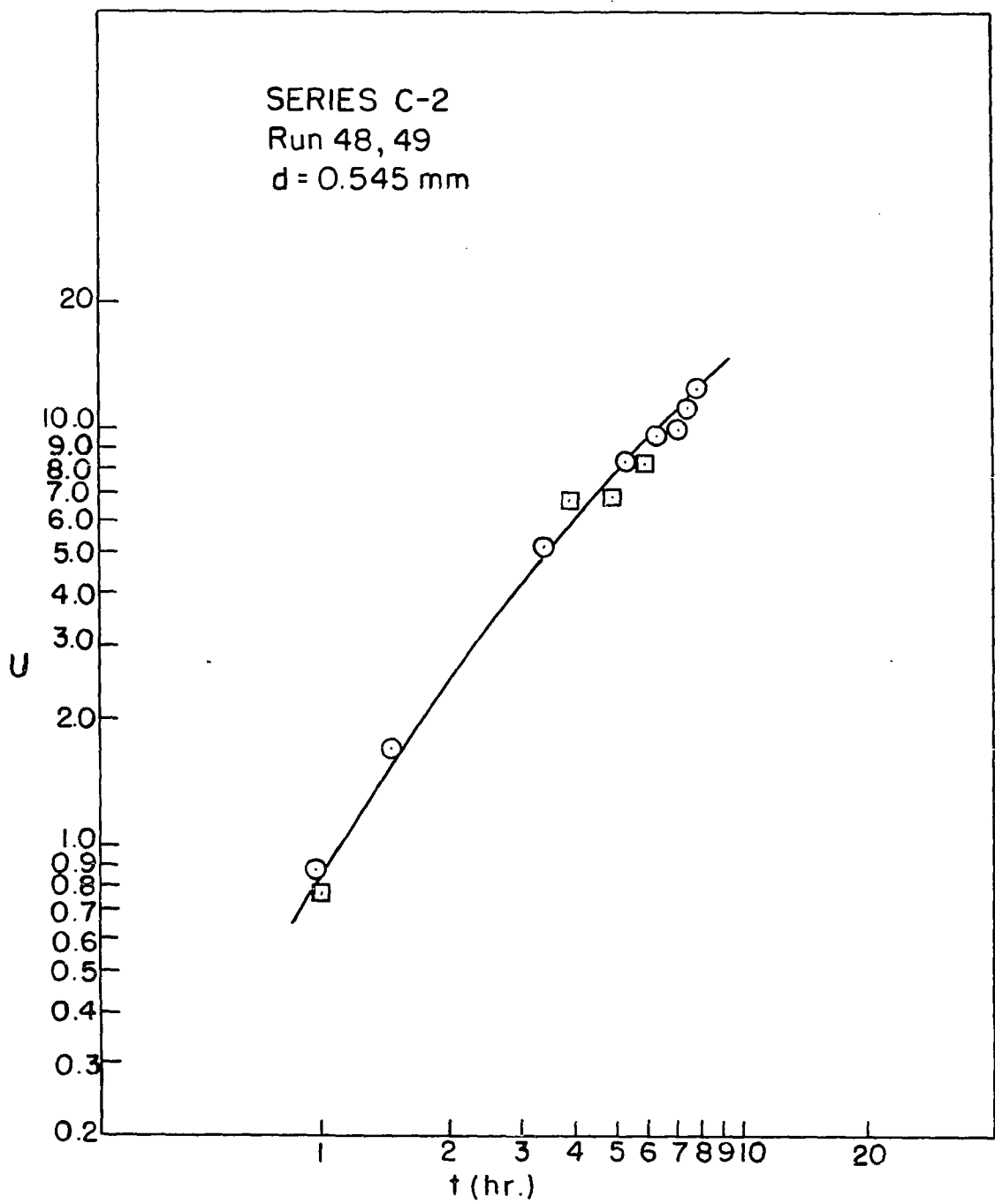


Figure 21. U vs filtration time for  $d = 0.458$  mm

$$Q = 6.0 \text{ gpm/sq ft}$$

$$\bar{C}_o = 3.4 \text{ mg/l}$$

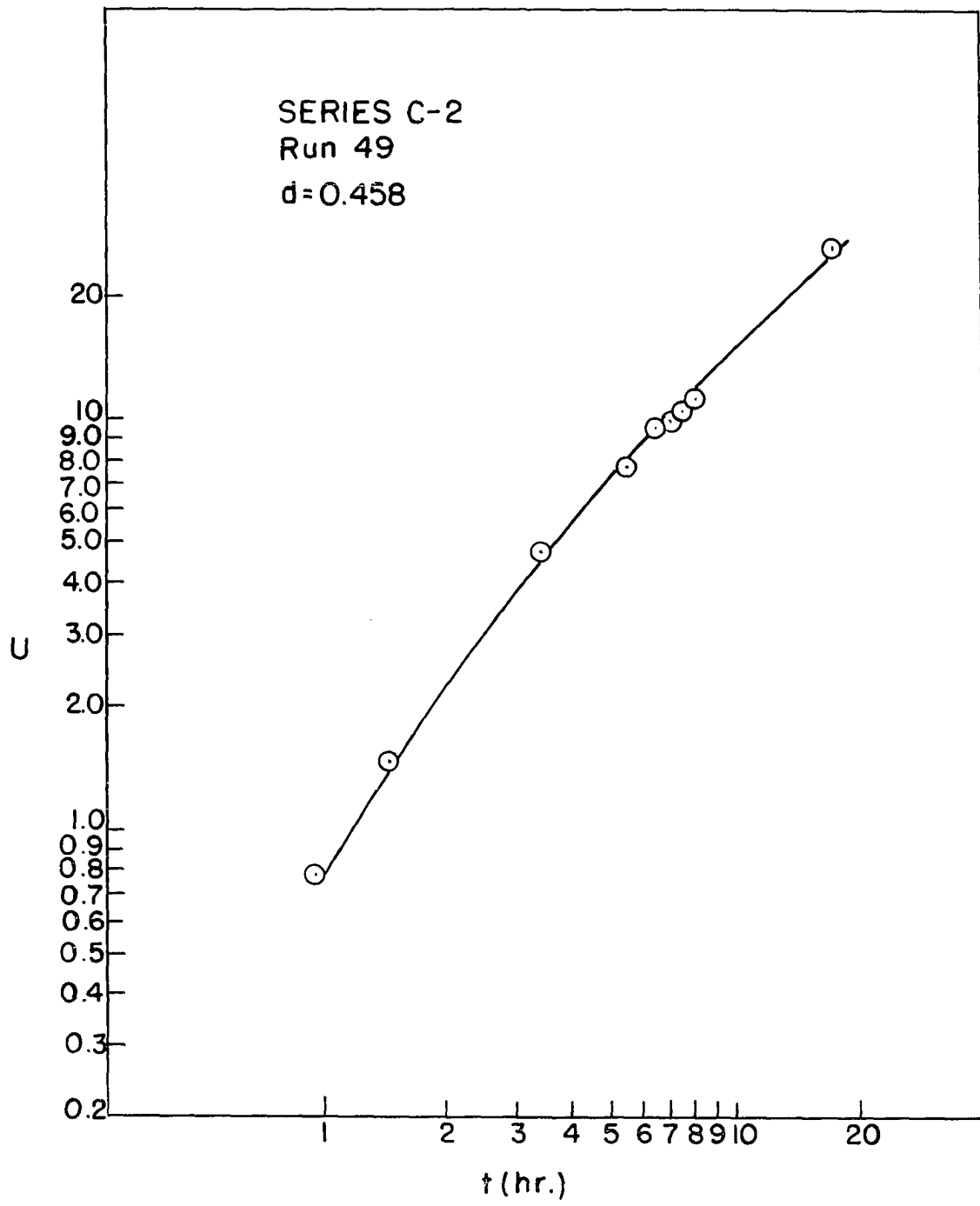


Figure 22. U vs filtration time for  $d = 0.386$  mm

$$Q = 6.0 \text{ gpm/sq ft}$$

$$\bar{C}_0 = 3.4 \text{ mg/l}$$

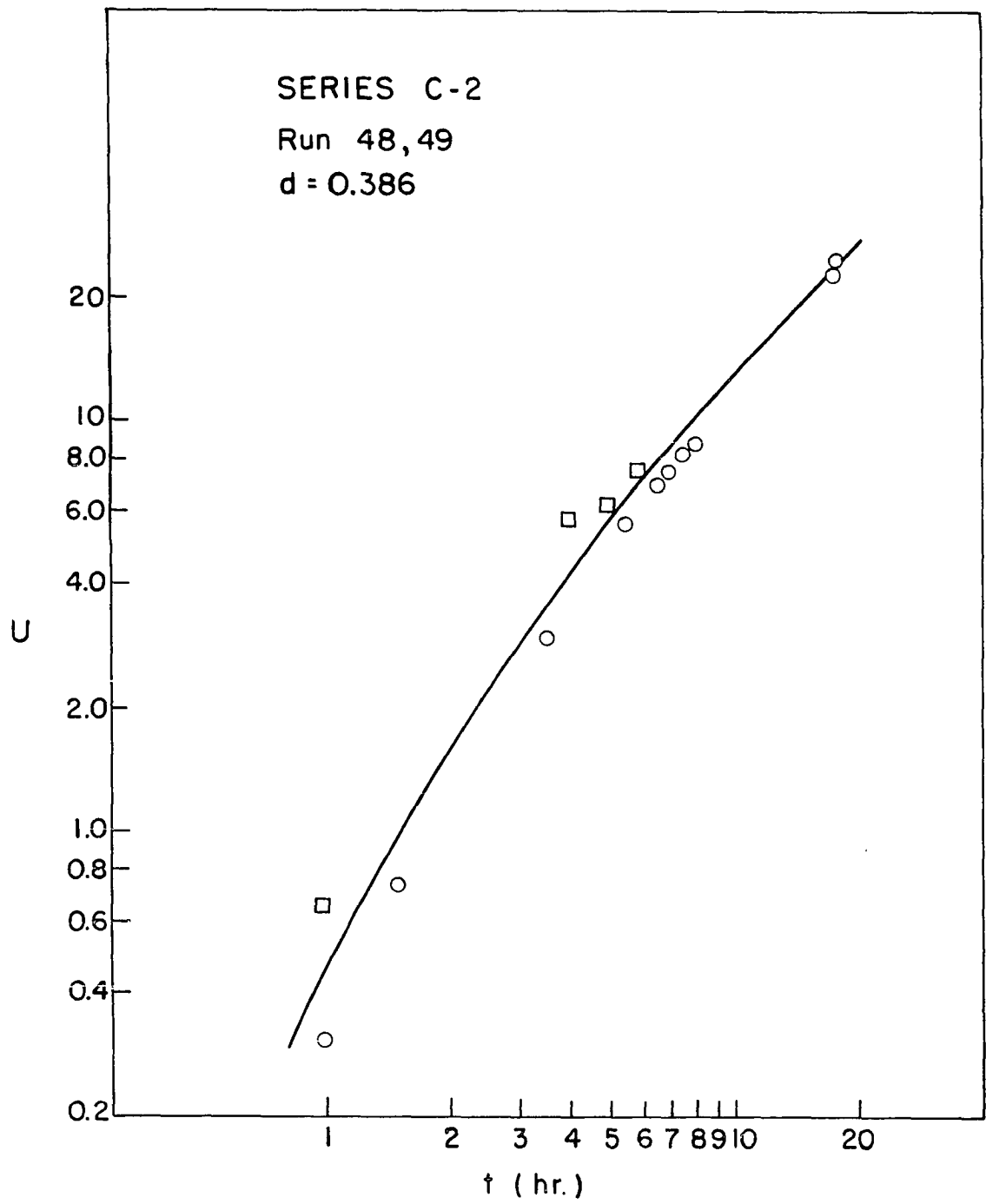


Figure 23. Relation of grain sizes to filtration time for constant U

$Q = 6.0 \text{ gpm/sq ft}$

$L = 1 \text{ in.}$



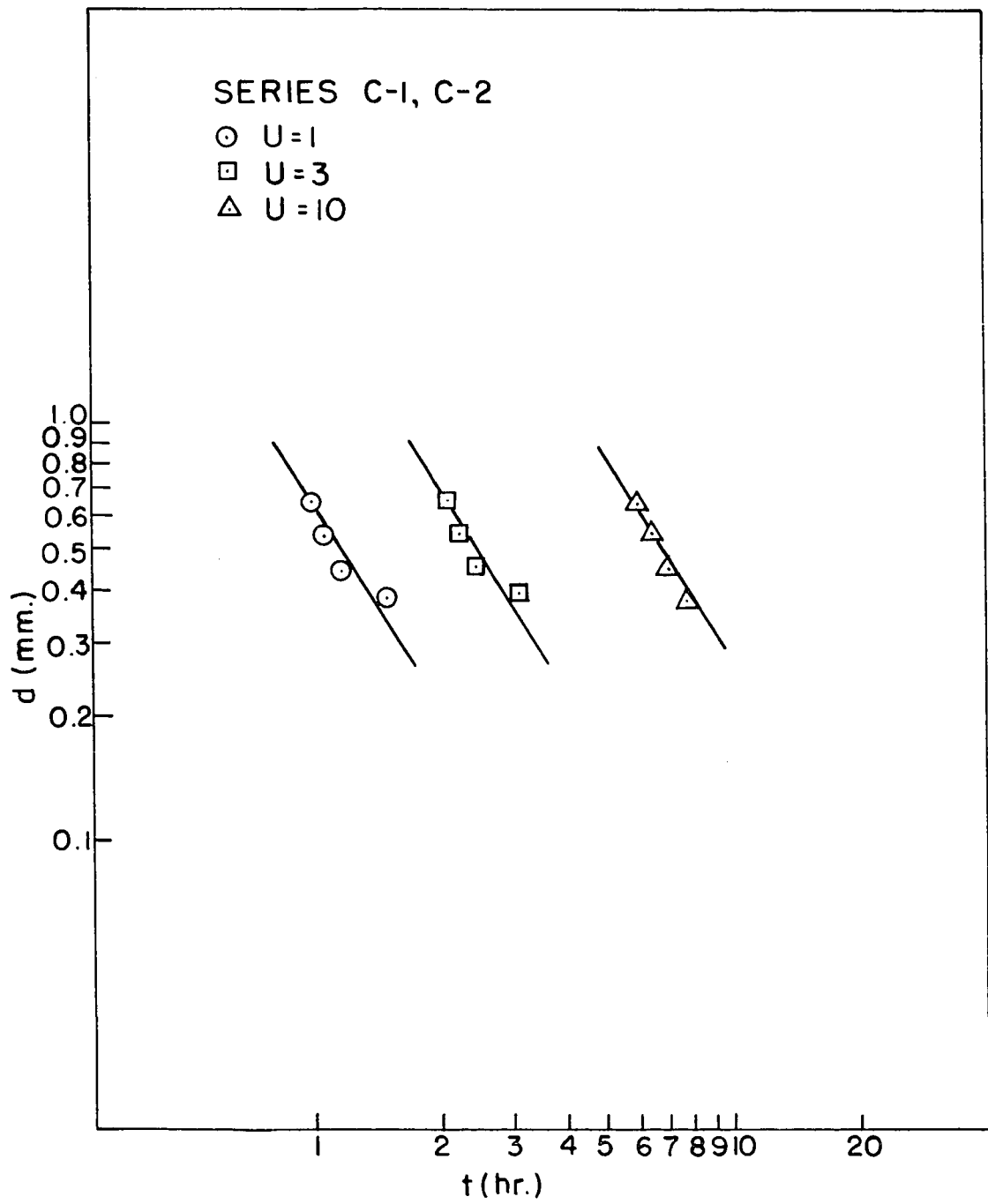


Figure 24. Run 42, 44, 60 and 62, U vs increase of head-loss

$$d = 0.649 \text{ mm}$$

$$Q = \begin{array}{l} 3.0 \text{ gpm/sq ft for Run 42} \\ 4.5 \text{ gpm/sq ft for Run 44} \\ 6.0 \text{ gpm/sq ft for Runs 60 and 62} \end{array}$$

$$\bar{C}_o = 3.4 \text{ mg/l}$$

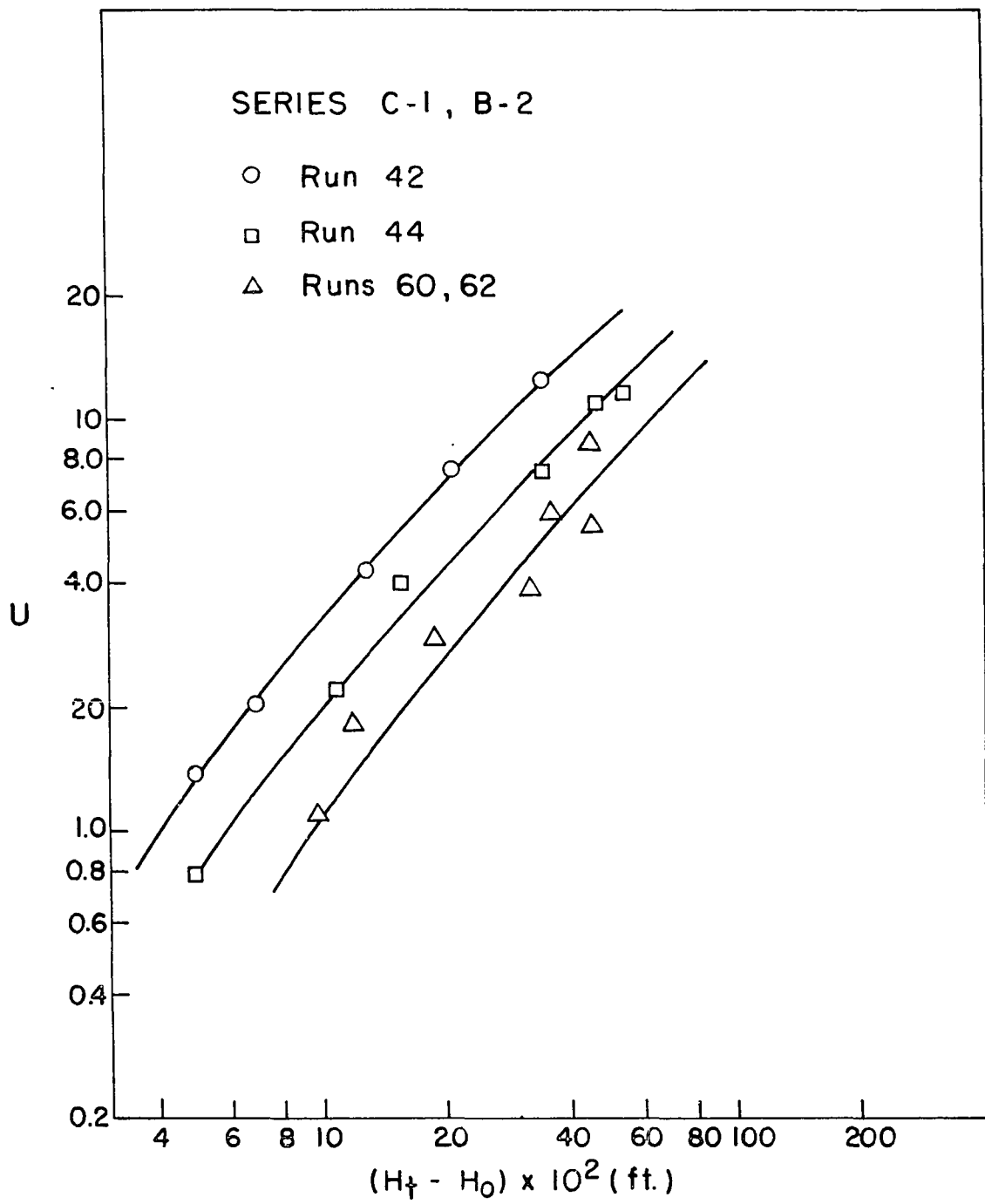


Figure 25. Relation of increase of head-loss to flow rate for constant U

$$d = 0.649 \text{ mm}$$

$$L = 1 \text{ in.}$$

$$\bar{C}_O = 3.4 \text{ mg/l}$$

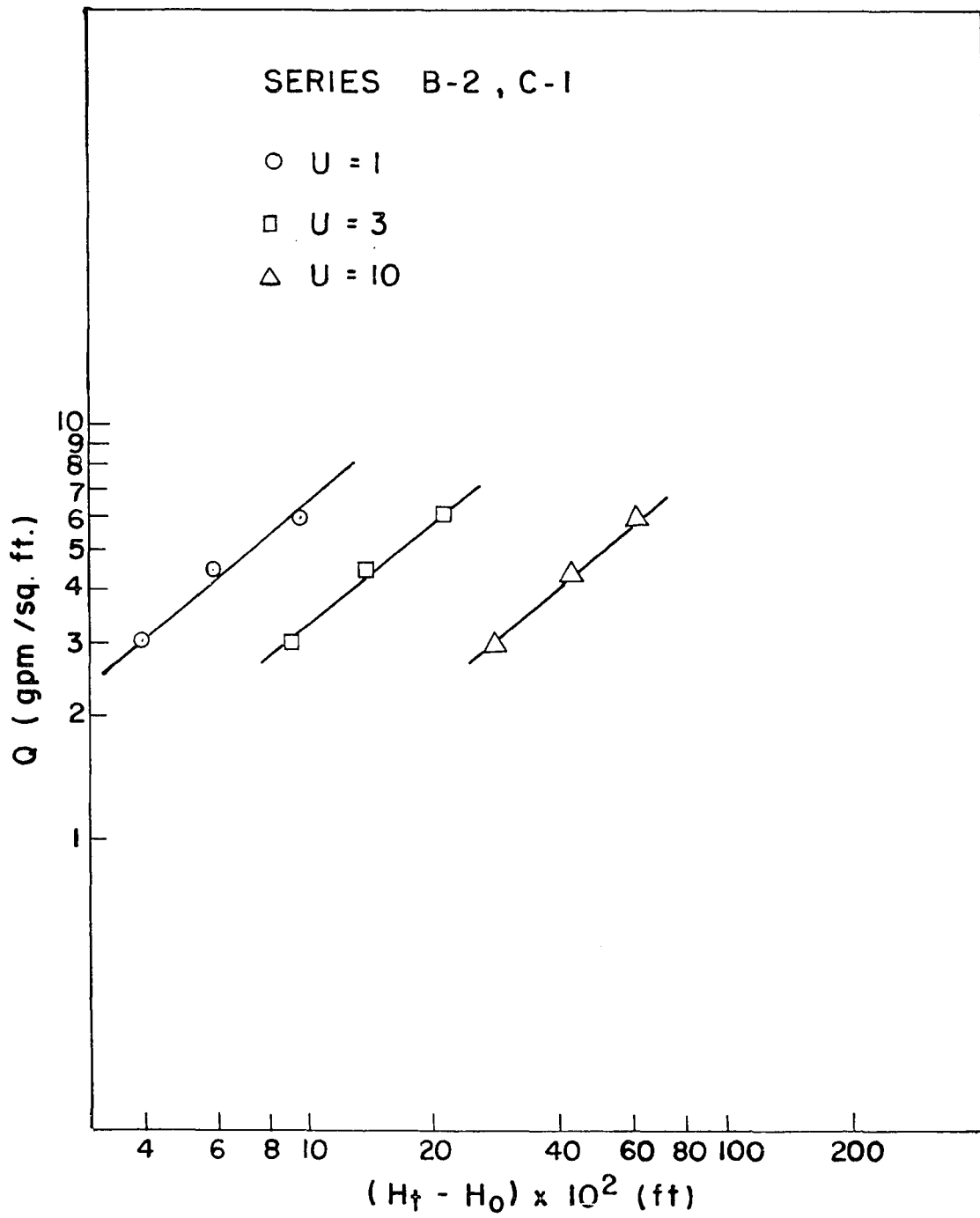


Figure 26. Runs 42 and 50, U vs increase of head-loss for different grain sizes

$$Q = 3.0 \text{ gpm/sq ft}$$

$$L = 1 \text{ in.}$$

$$\bar{C}_o = 3.3 \text{ mg/l}$$

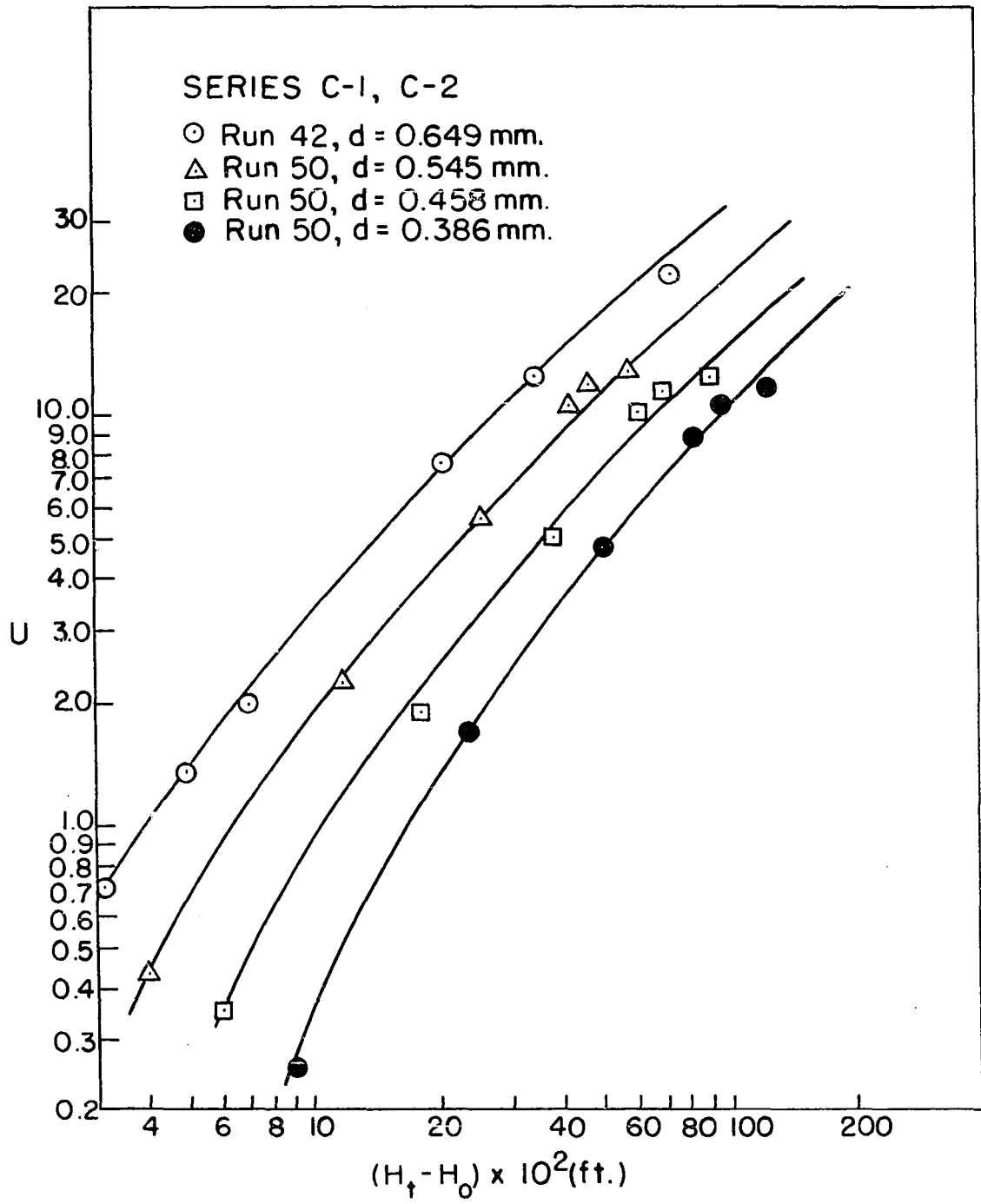


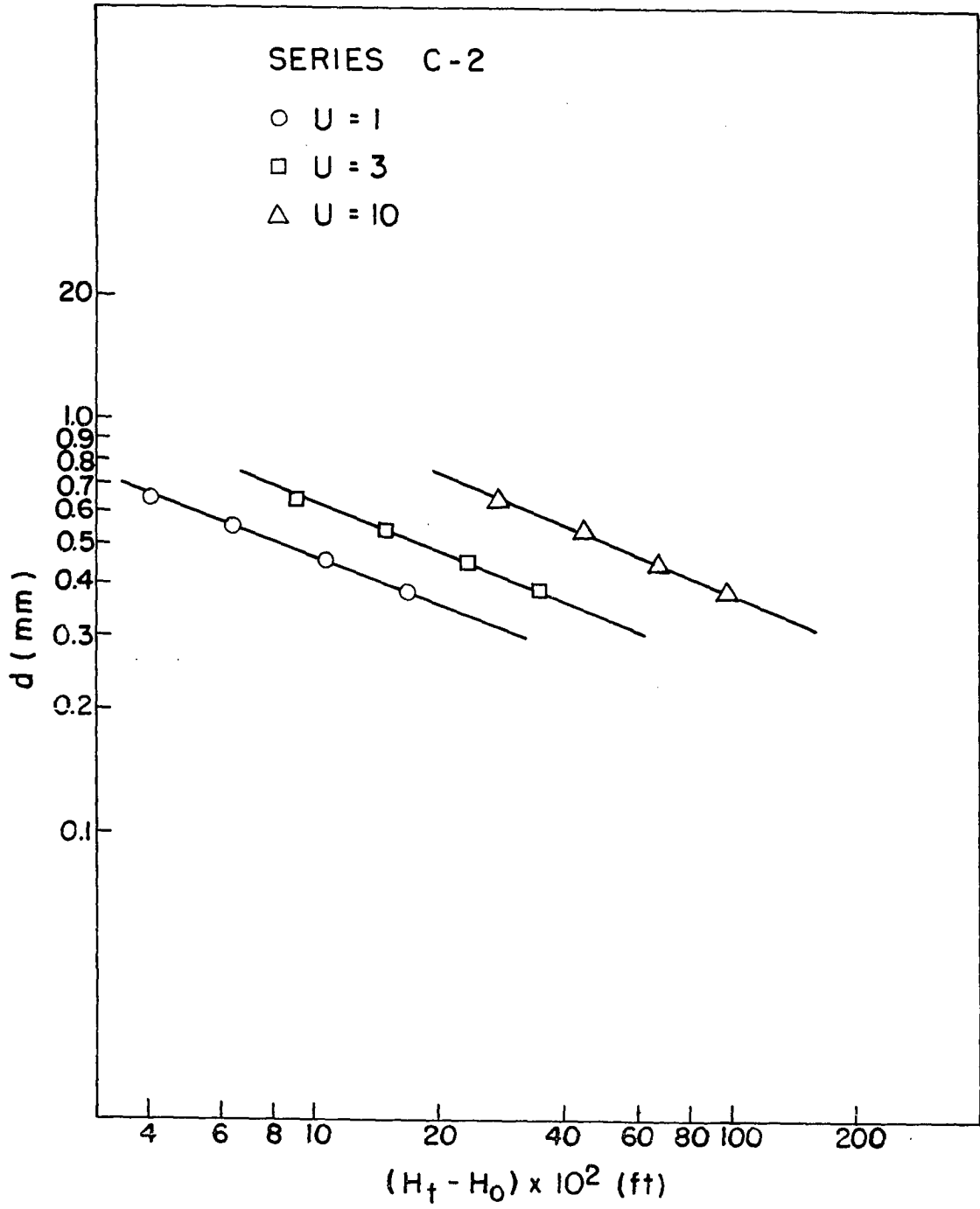
Figure 27. Relation of grain size to increase of head-loss for constant U

$$Q = 3.0 \text{ gpm/sq ft}$$

$$L = 1 \text{ in.}$$

$$\bar{C}_0 = 3.3 \text{ mg/l}$$





## 6. Effect of influent concentration on head-loss

Figures 28 through 30 show the effect of influent concentration on head-loss. For a given U value, the slope of the plot for  $C_o$  vs  $(H_t - H_o)$  is approximately equal to  $1/1.4$  as shown in Figure 31; therefore  $\log C_o \propto (\alpha/\alpha.4) \log (H_t - H_o)$ , or  $(H_t - H_o) \propto C_o^{1.4}$ , and for given value of  $(H_t - H_o)/C_o^{1.4}$ , the U is constant for the given depth.

## 7. Derivation of grouped term G

From the above analyses on the effect of the three variables, Q, d and t on effluent quality, it has been noted that for a given value of  $Q^{0.29}t$  or  $d^{0.62}t$ , the U value is constant as mentioned on pages 80 and 89. Such variables have been grouped into one term G as follows:

$$G = Q^{a_1} d^{a_2} t \quad \text{Eq 15}$$

where the exponentials  $a_1$  and  $a_2$  are equal to 0.29 and 0.62 respectively under the experimental conditions of this study.

## 8. Derivation of grouped term R

The effects of flow rate, grain size and influent concentration on the head-loss have been discussed on page 89 and in the previous sub-section 6. For a given value of

$(H_t - H_o)/Q^{1.2}$ ,  $(H_t - H_o)/C_o^{1.4}$ , or  $(H_t - H_o)d^{2.5}$ , the U value is constant. Thus, the four variables  $(H_t - H_o)$ , q, d, and  $C_o$

have been grouped into one term R in the following expression:

$$R = d^{b_1} (H_t - H_o) / Q^{b_2} C_o^{b_3} \quad \text{Eq 16}$$

where the exponentials  $b_1$ ,  $b_2$  and  $b_3$  equal to 2.5, 1.2 and 1.4 respectively under the experimental conditions of this study.

Figure 28. U vs increase of head-loss for  
different influent concentrations

$$d = 0.649 \text{ mm}$$

$$Q = 6.0 \text{ gpm/sq ft}$$

$$L = 1 \text{ in.}$$

$$\bar{C}_o = \begin{array}{l} 5.8 \text{ mg/l for Runs 38, 39, 41, 45} \\ 4.5 \text{ mg/l for Run 64} \\ 3.4 \text{ mg/l for Runs 60 and 62} \end{array}$$

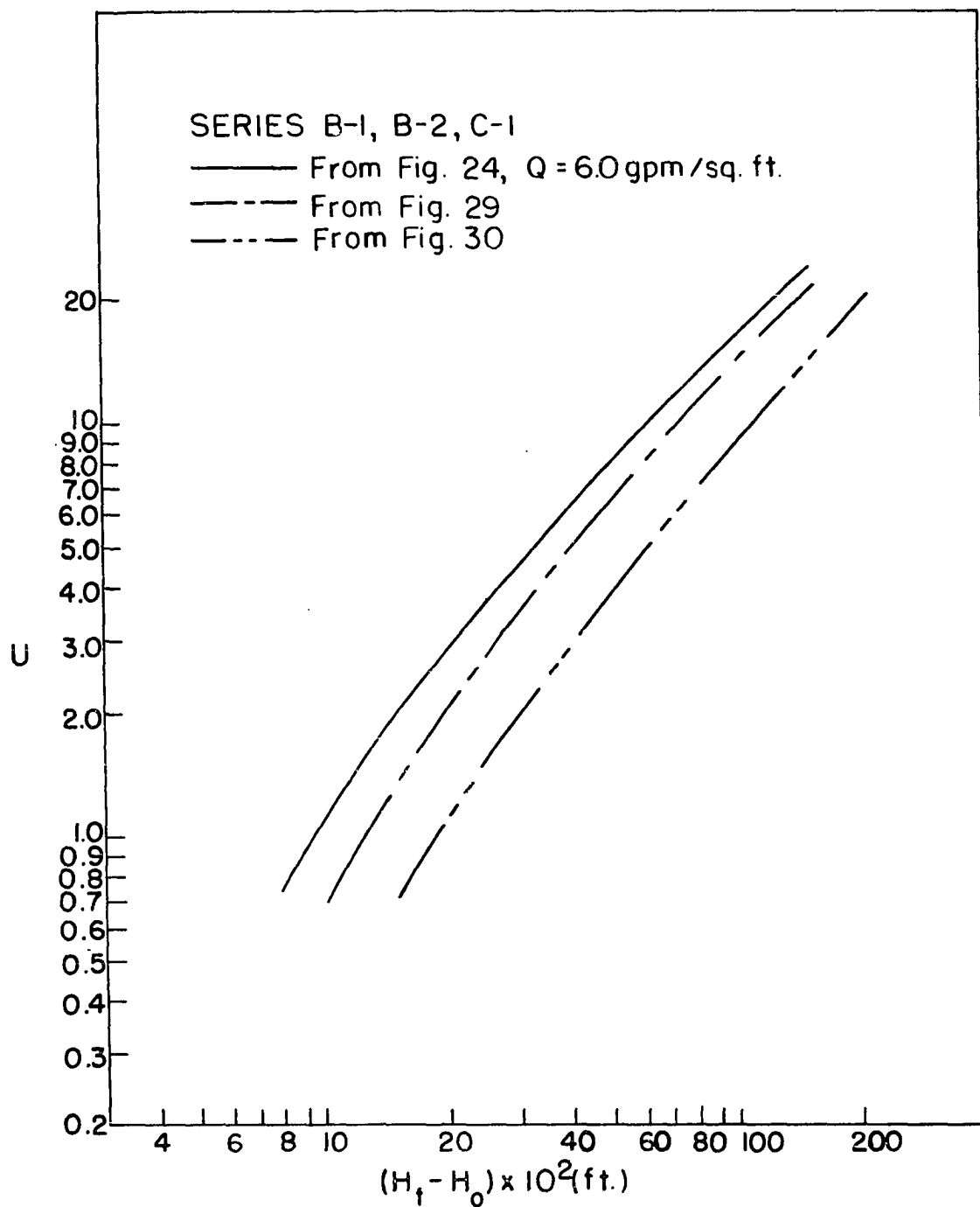


Figure 29. U vs increase of head-loss for  
 $C_o = 4.5 \text{ mg/l}$

$d = 0.649 \text{ mm}$

$Q = 6.0 \text{ gpm/sq ft}$

$L = 1 \text{ in.}$

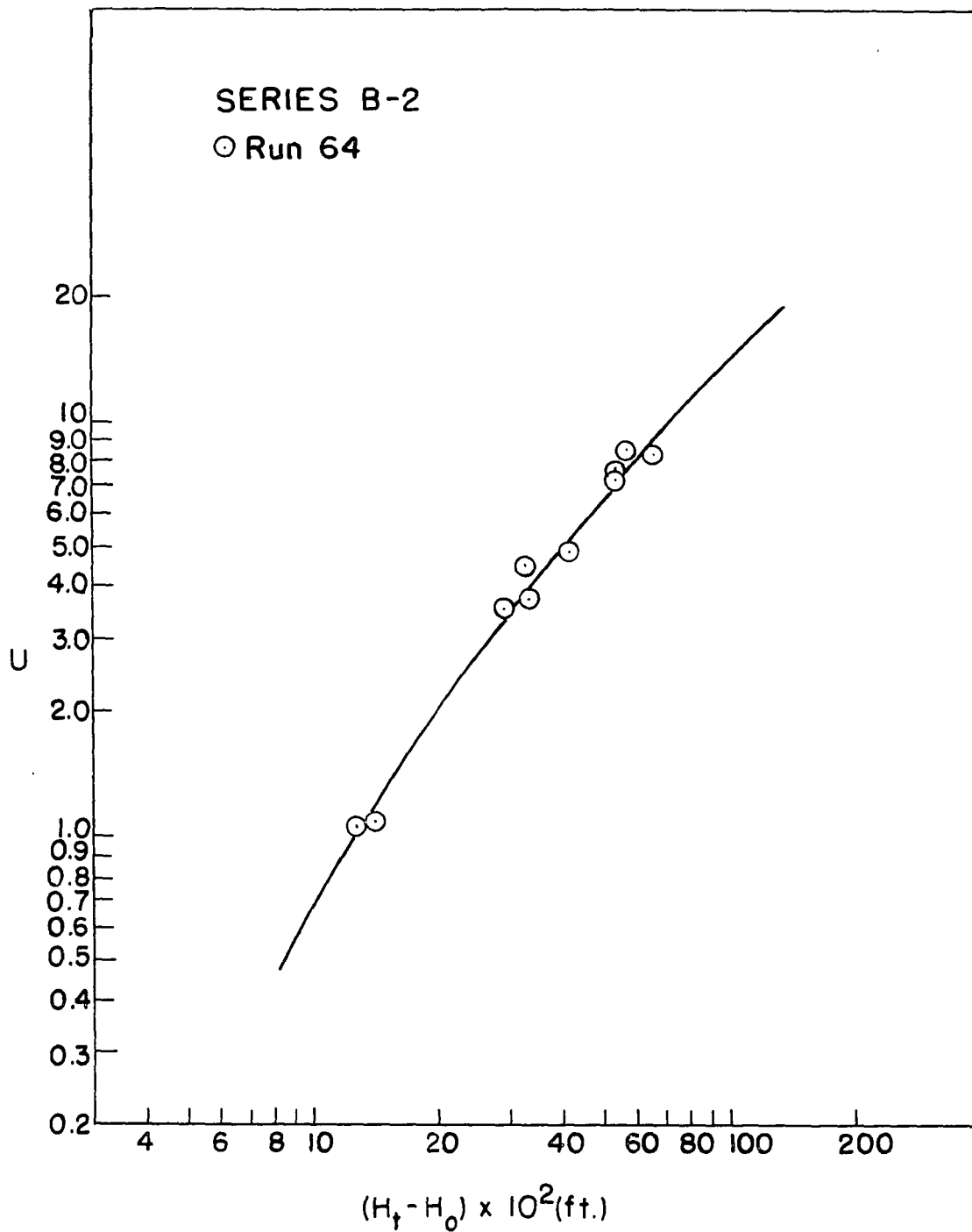


Figure 30. U vs increase of head-loss for  
 $C_0 = 5.8 \text{ mg/l}$

$d = 0.649 \text{ mm}$

$Q = 6.0 \text{ gpm/sq ft}$

$L = 1 \text{ in.}$

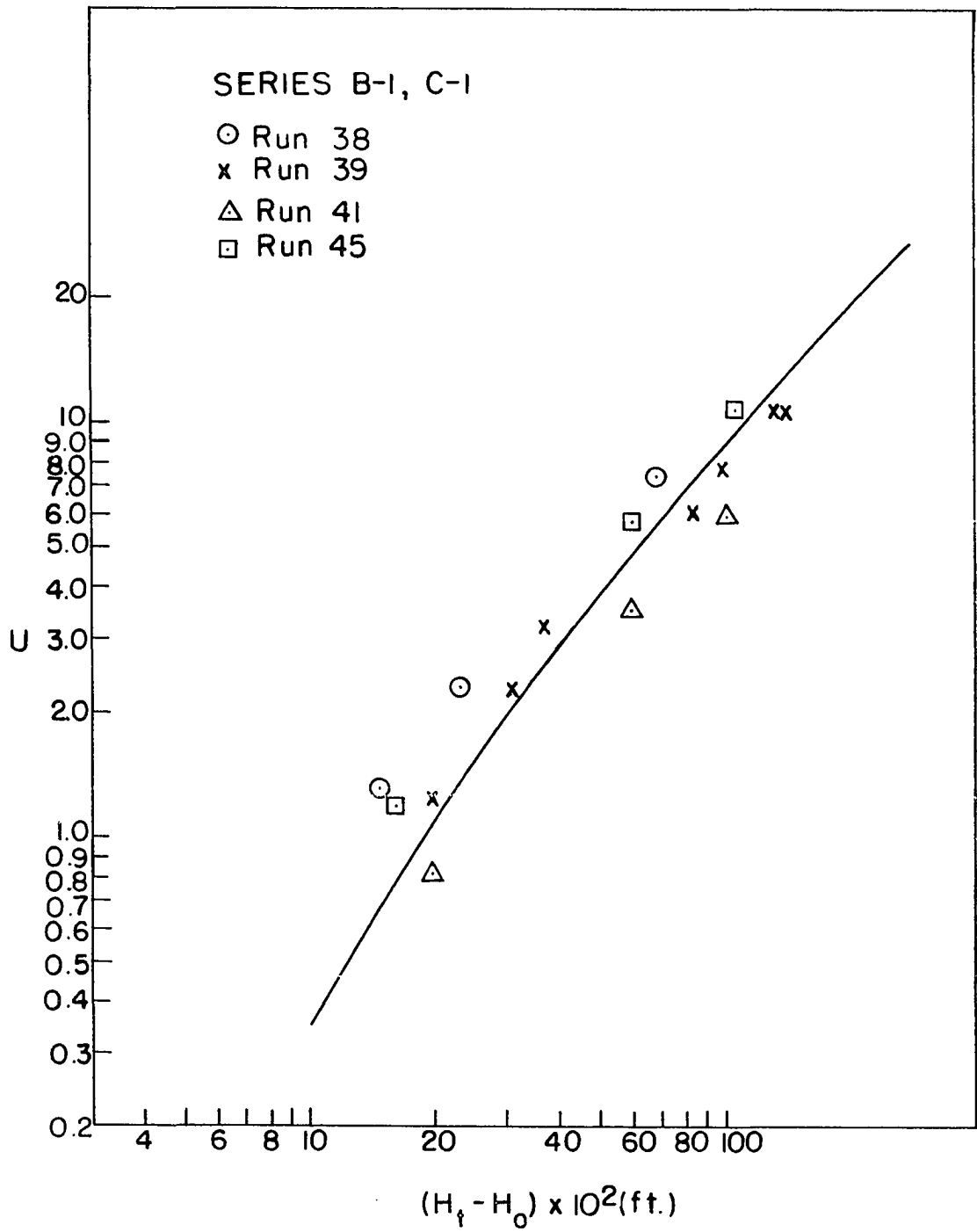


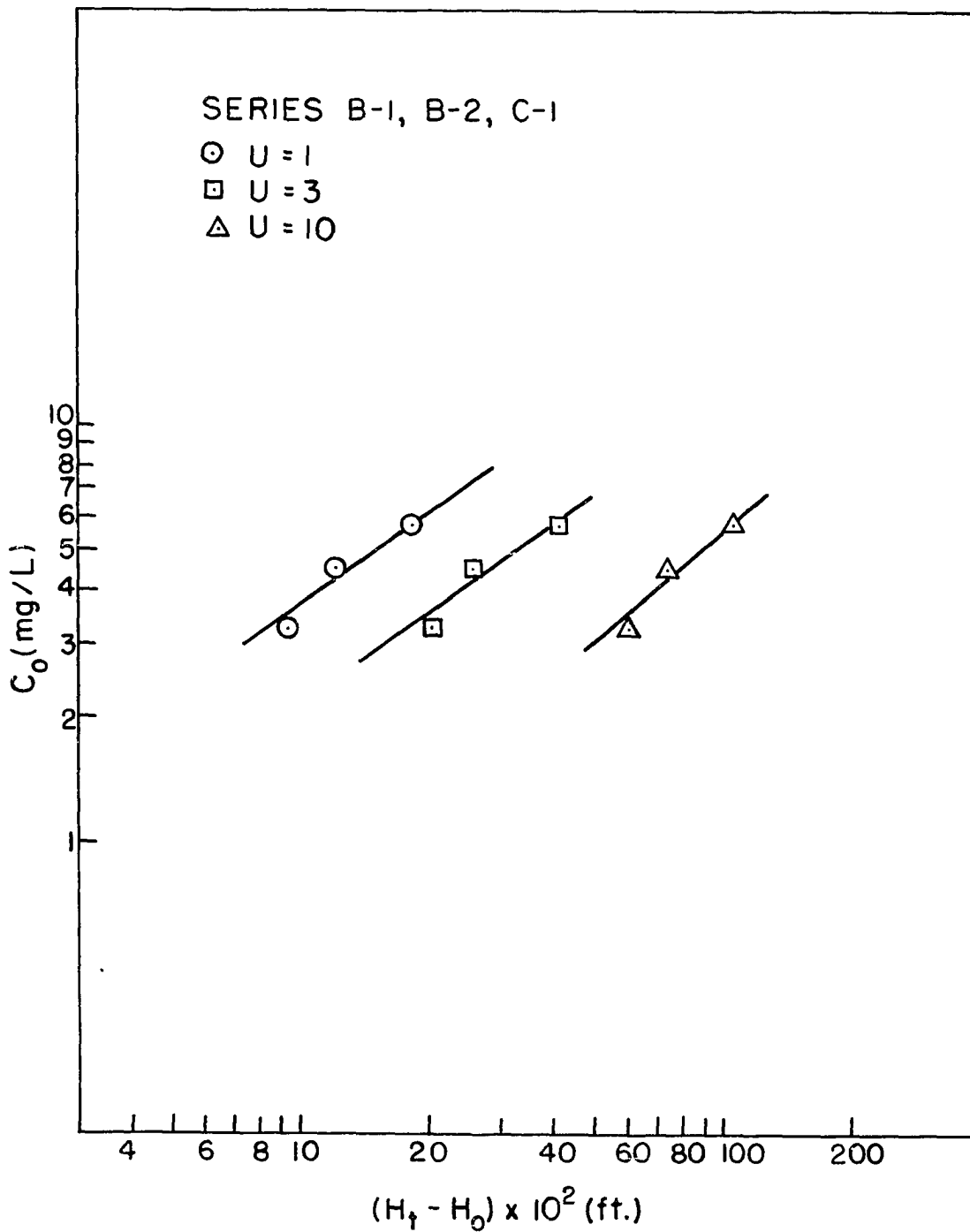


Figure 31. Relation of influent concentration to increase of head-loss for constant U

$$d = 0.649 \text{ mm}$$

$$Q = 6.0 \text{ gpm/sq ft}$$

$$L = 1 \text{ in.}$$



### C. The Relationship between U and Other Filtration Parameters

#### 1. U is a function of G

When U is plotted against G, the data from all experimental runs can be lumped together and a family of curves similar to that in Figures 8 and 10 have been obtained as shown in Figures 32, 33 and 34 in which the curves have been fitted by multiple regression. Figure 35 is a plot for equi-U curves similar to Figure 11, but using L and G as coordinates.

#### 2. U is a function of R

When U is plotted against R, similar curves have been obtained for the lumped data as shown in Figures 36, 37 and 38. The equi-U curves are plotted in Figure 39, using L and R as coordinates.

#### 3. U is a function of specific deposit, $\sigma$

It has been found that U is definitely correlated with the specific deposit,  $\sigma$ , which can be calculated based on equations 1 and 2. The method of calculation of  $\sigma$  has been described in detail by Ives (1960a), and by Fox and Cleasby (1966). An example of such calculation is presented in Table 12 in the Appendix.

When the U value corresponding to a given depth, time and filtrate concentration are plotted against the corresponding  $\sigma$  on log-log paper, a good linearity is noted within the range of investigation as shown in Figure 40. Such linearity is not affected by the choice of the units used to express  $\sigma$ .

Figure 32. U vs G for different depths, lumped data for various grain sizes, flow rates and influent concentrations

$d = 0.386, 0.485, 0.545, 0.649 \text{ mm}$

$Q = 3.0, 4.5, 6.0 \text{ gpm/sq ft}$

$\bar{C}_o = 3.3 \text{ and } 5.8 \text{ mg/l}$

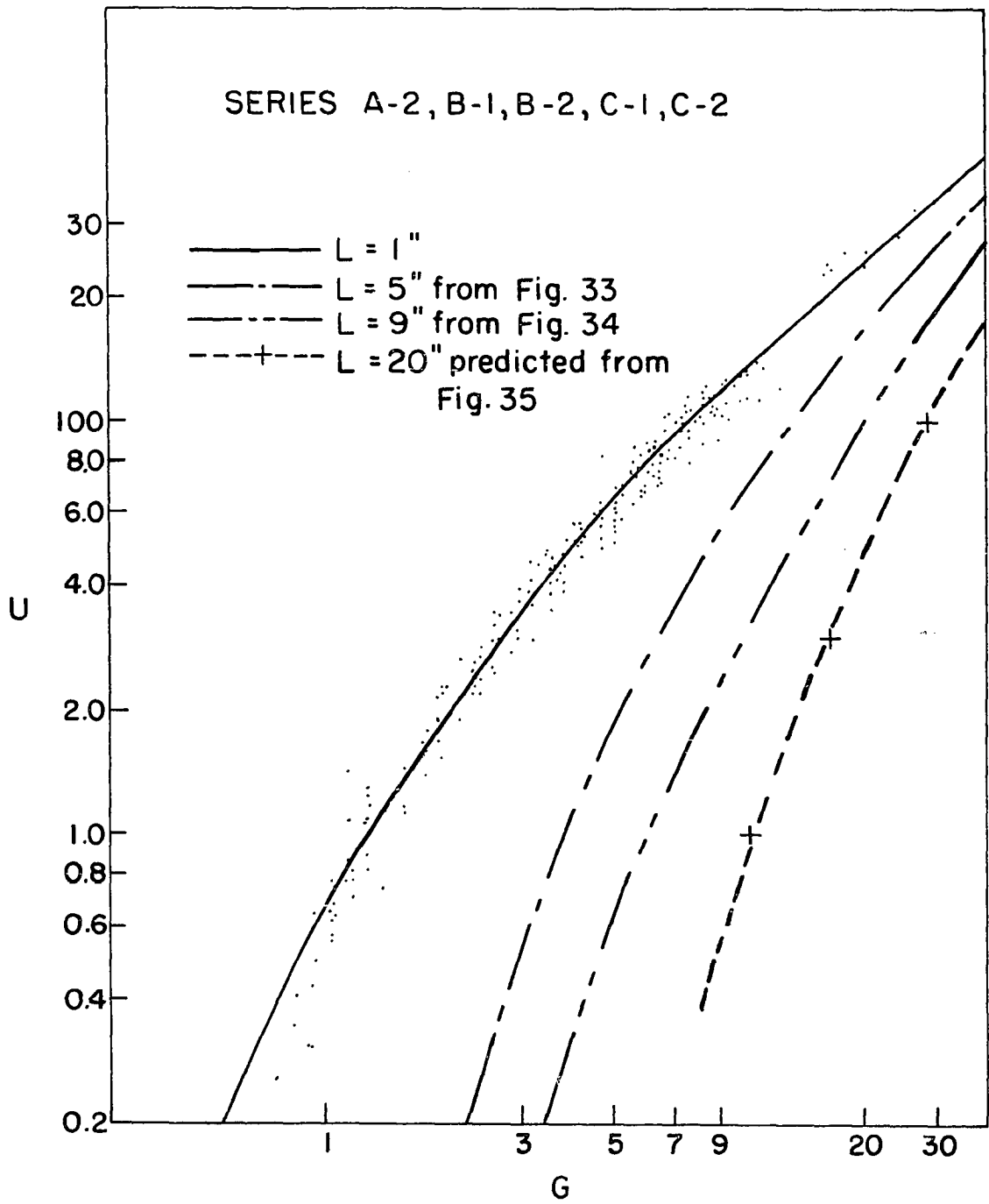


Figure 33. U vs G for L = 5 in., lumped data  
for various grain sizes, flow rates  
and influent concentrations

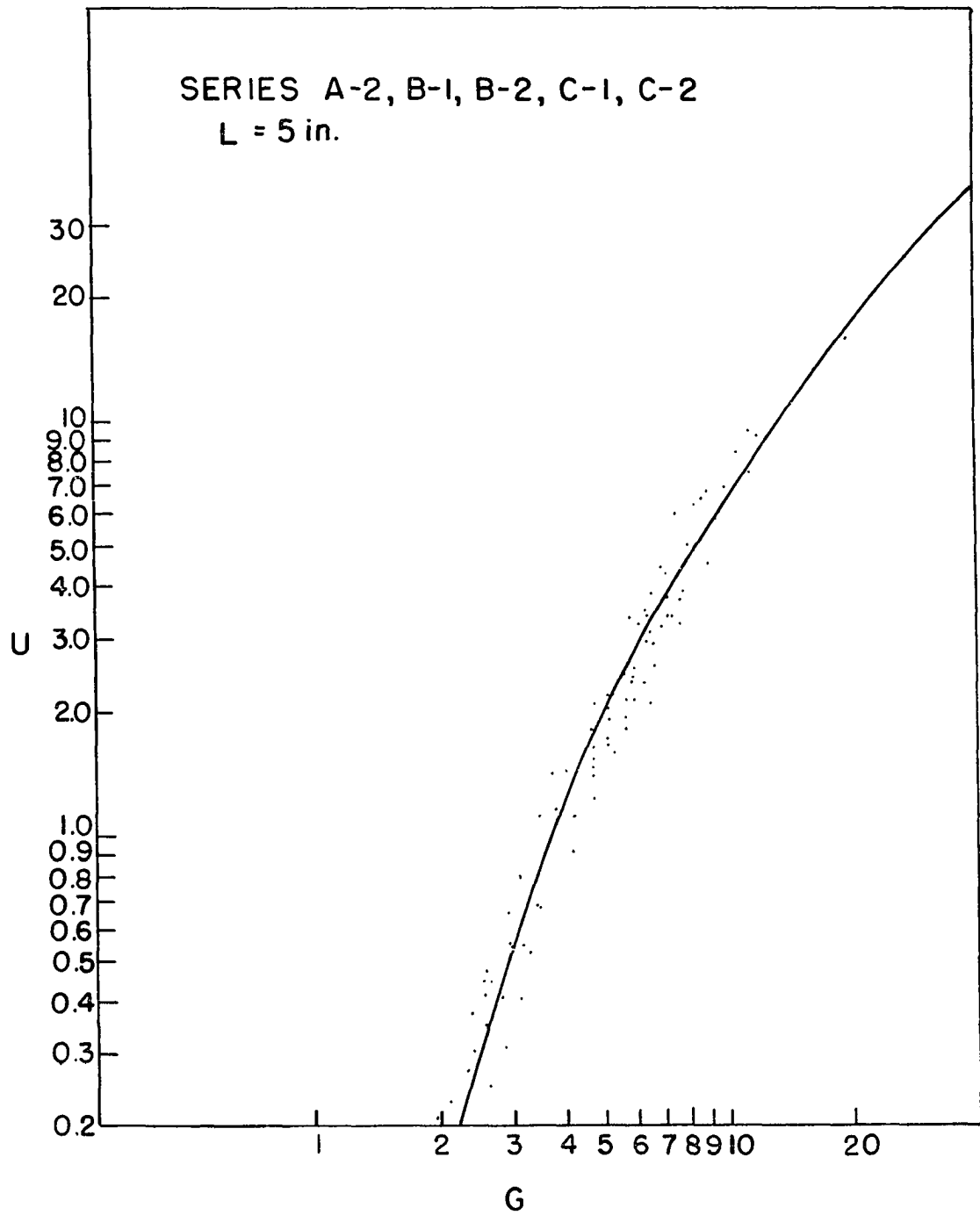


Figure 34. U vs G for L = 9 in., lumped data  
for various grain sizes, flow rates  
and influent concentrations



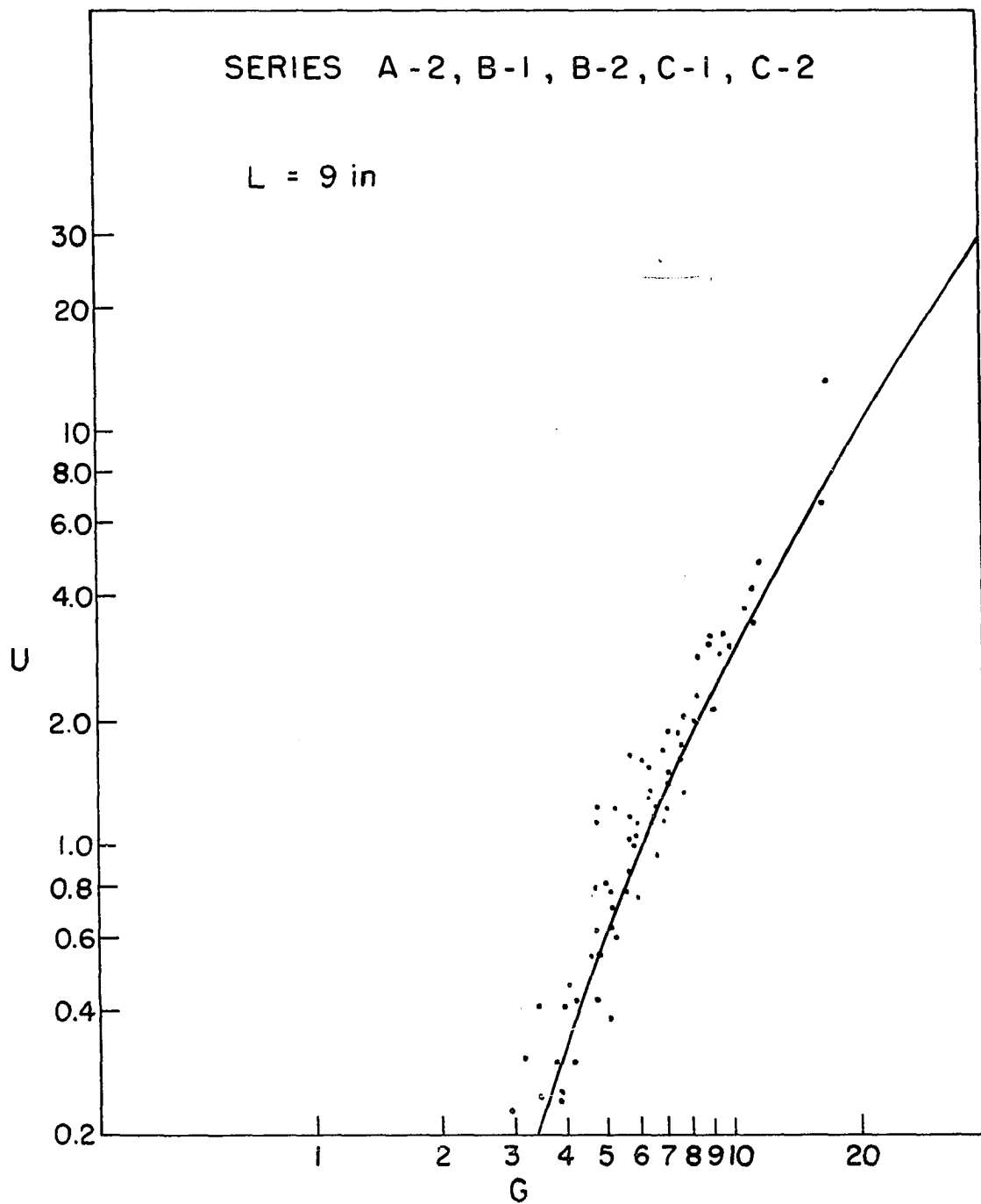


Figure 35. Equi-U curves for L vs G, lumped data for various grain sizes, flow rates and influent concentrations

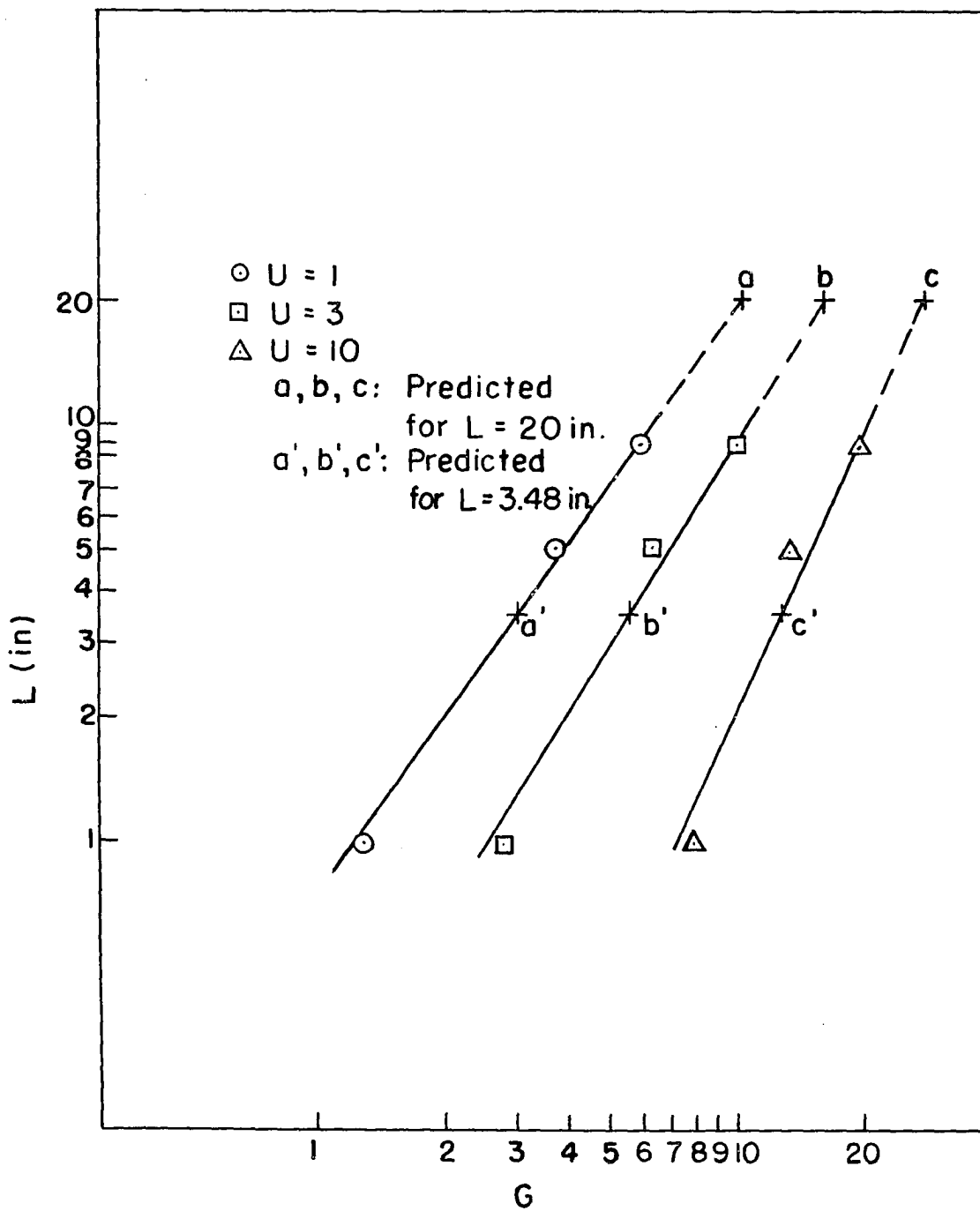


Figure 36. U vs R for different depths, lumped data for various grain sizes, flow rates and influent concentrations

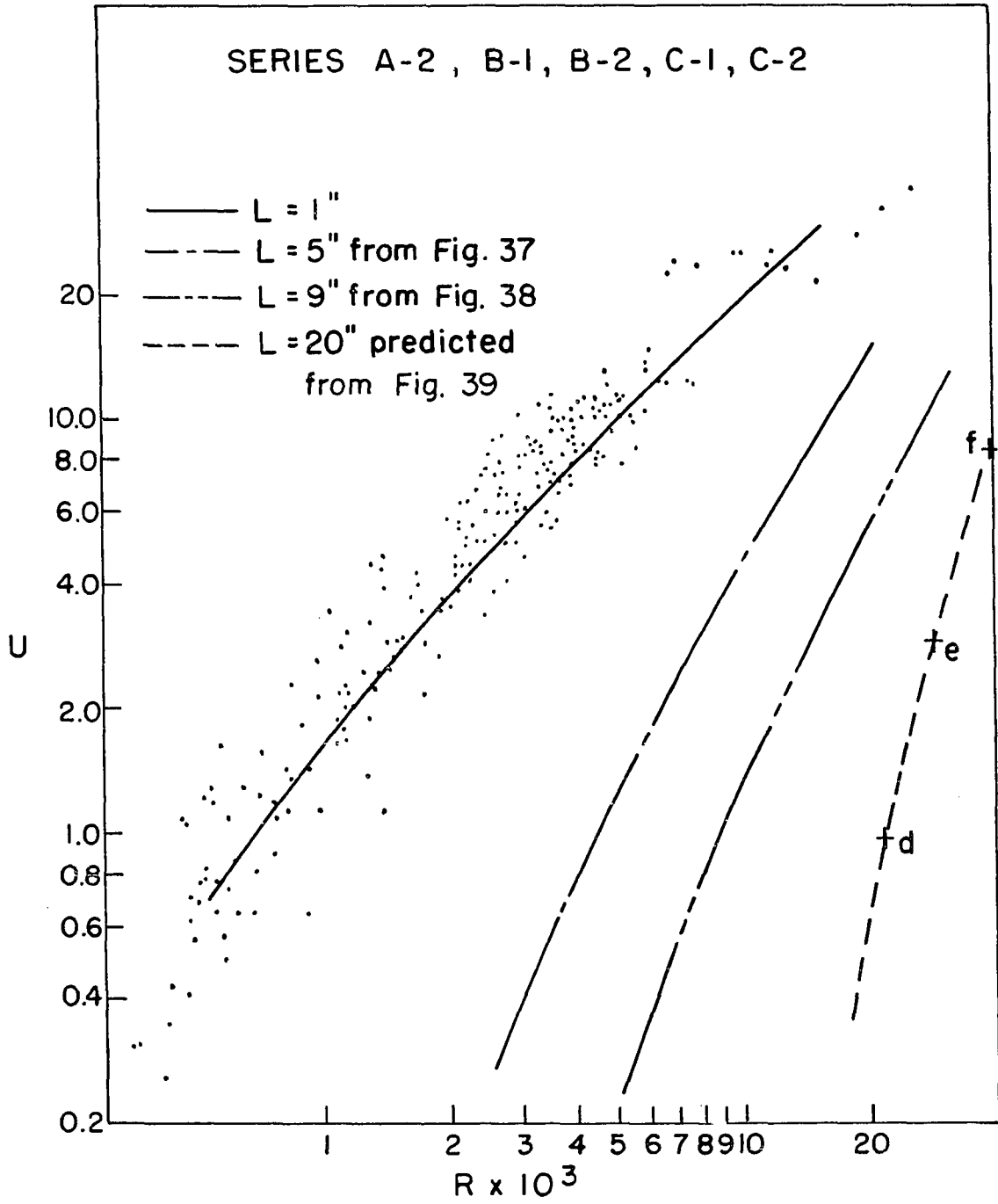


Figure 37. U vs R for L = 5 in., lumped data  
for various grain sizes, flow rates  
and influent concentrations

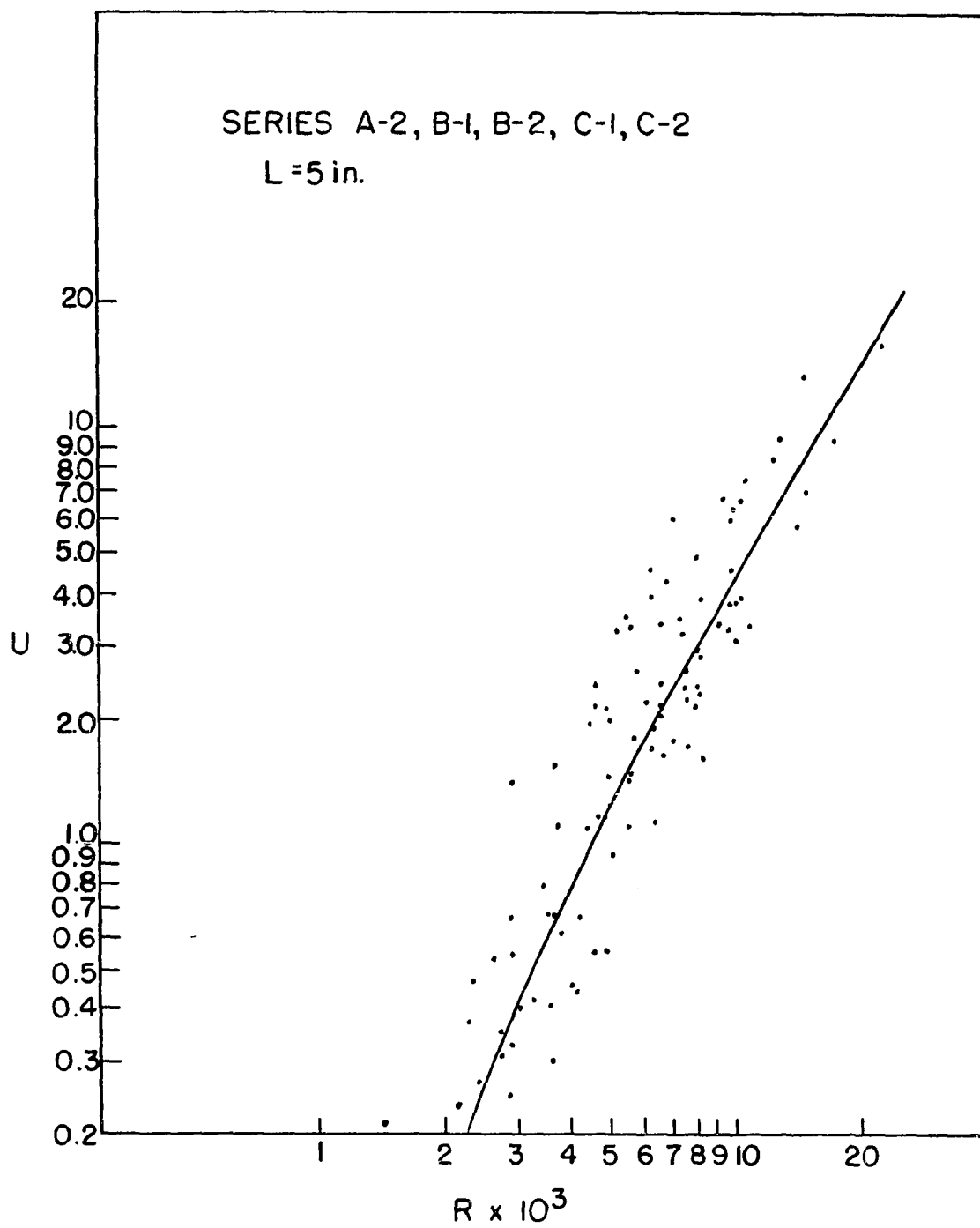


Figure 38. U vs R for L = 9 in., lumped data  
for various grain sizes, flow rates  
and influent concentrations



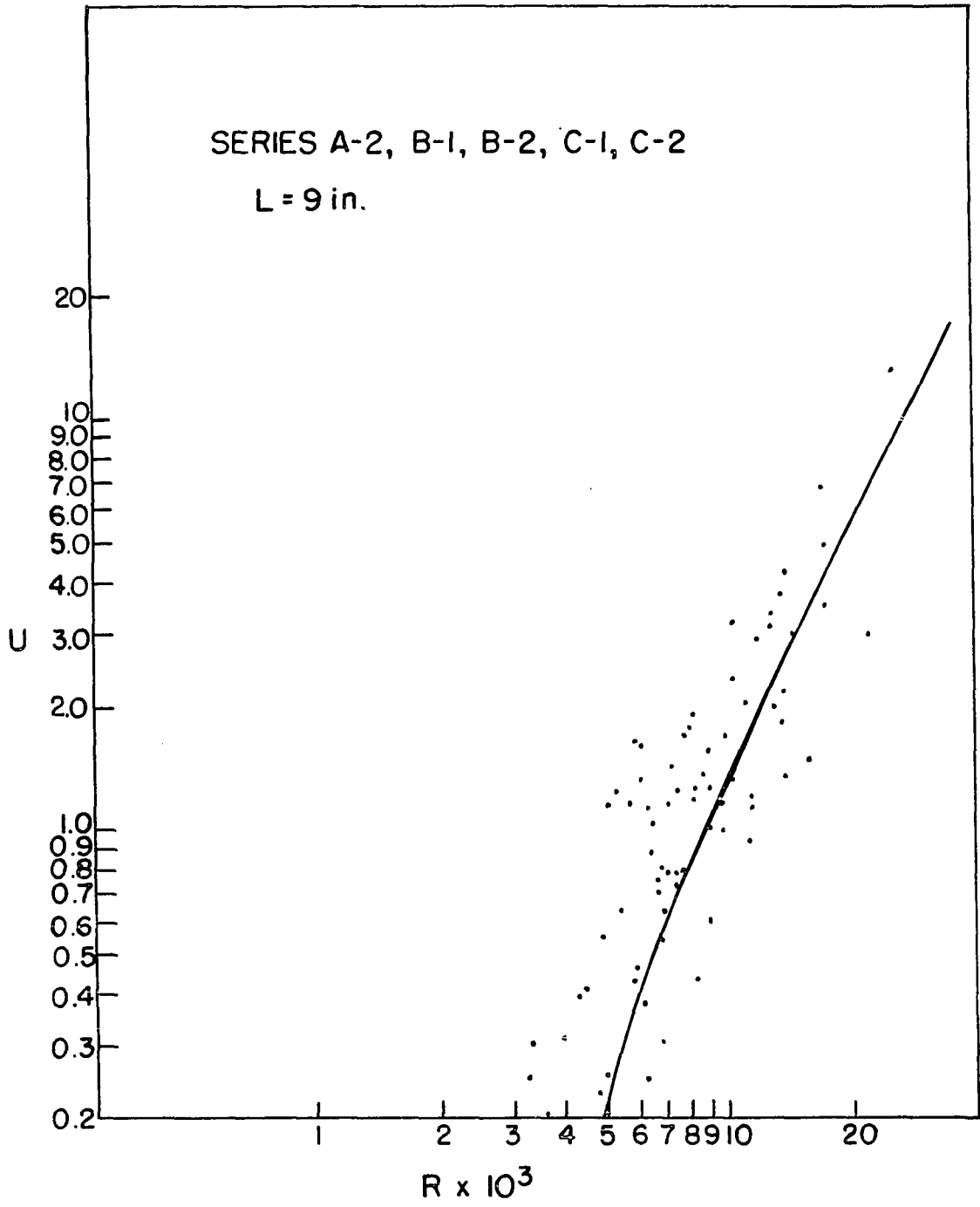


Figure 39. Equi-U curves for L vs R, lumped data for various grain sizes, flow rates and influent concentrations

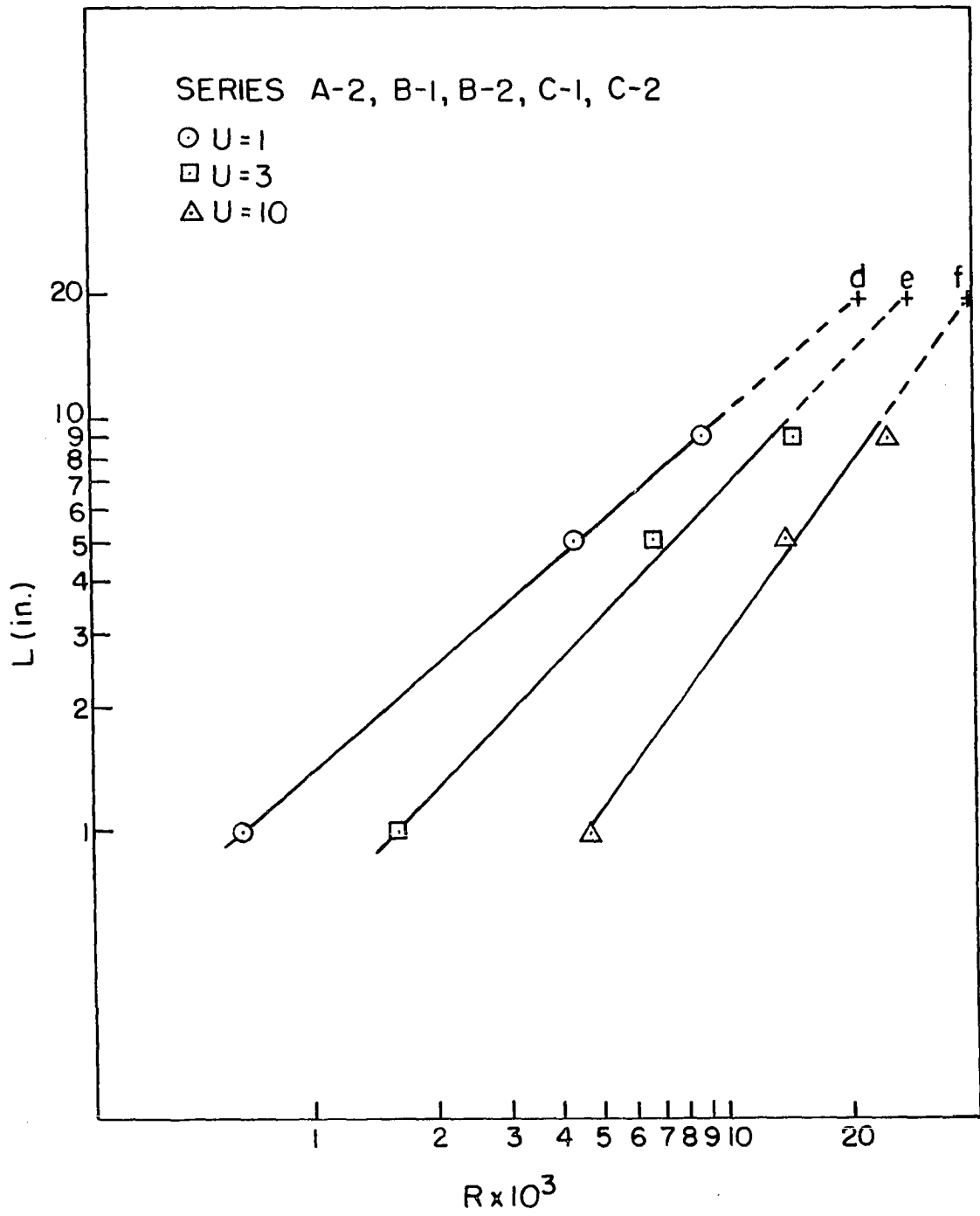
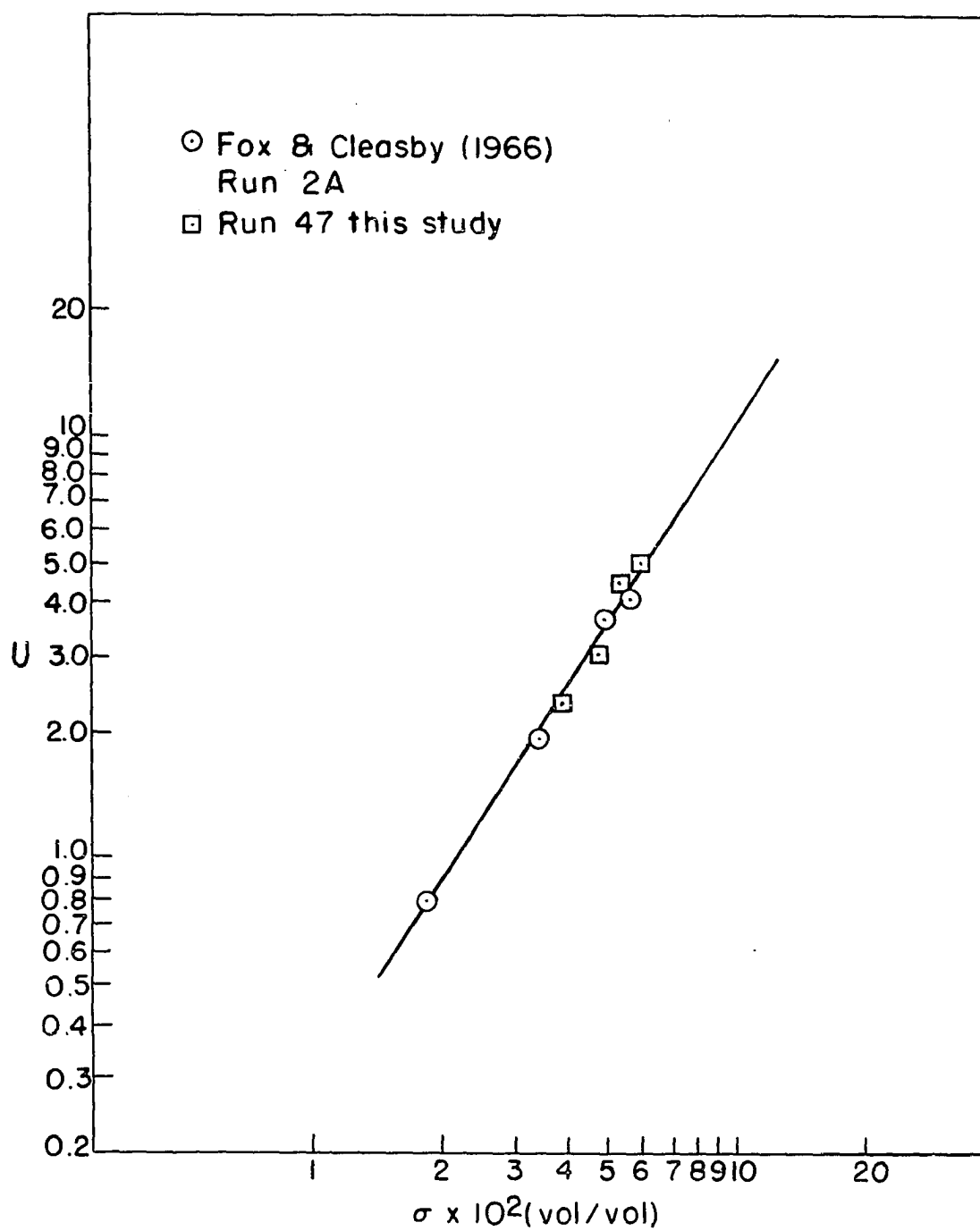


Figure 40. Relation of U to specific deposit ( $\sigma$ )

Fox & Cleasby (1966) Run 2A	This Study Run 47
$d = 0.705$ mm	$d = 0.649$ mm
$Q = 3.86$ gpm/sq ft	$Q = 4.5$ gpm/sq ft
$L = 3.7$ cm	$L = 2.54$ cm
$\bar{C}_o = 5.5$ mg/l (iron)	$\bar{C}_o = 6.0$ mg/l (iron)
$T = 15.6$ °C	$T = 25.0$ °C



However, the absolute position of the curve would be altered. Because of the relation between  $U$  and  $\sigma$ ,  $U$  may be tentatively named the "Deposit Index" and will be referred to by this name hereinafter in this study.

#### D. Performance Curves

It has been noted that the degree of curvature of the curves in Figures 32 and 36 vary with depths; the deeper the depth the flatter the curve. By proper shifting of the curves for depths of 5 and 9 in. first downwardly and then to the left relative to the position of the curve for 1 in. depth, it is possible to form two single curves, one for Figure 32 and one for Figure 36. These will be called performance curves. By a few trials this can be achieved by dividing  $U$  by  $L$  and dividing  $G$  and  $R$  by  $L^{a_3}$  and  $L^{b_4}$  respectively. Under the experimental conditions of this study, a value of  $a_3 = 1.2$  and  $b_4 = 1.6$ , result in the desired shifting of the curves in Figures 32 and 36 respectively.

It can probably be expected that all of the exponentials used in developing the performance curves may vary with the type of influent suspension and/or the filter media, and should be evaluated accordingly. However, the method of developing the performance curves should be applicable to all suspensions and filter media.

##### 1. Performance Curve I

When  $U/L$  is plotted vs  $G/L^{1.2}$  for the lumped data of

various grain size, flow rate, concentration, depth of bed and time of filtration, a single curve to best fit the plotted data can be obtained either by visual best fit or by the method of multiple regression analysis. The latter method was used in this study.

The multiple regression equation can be denoted by

$$\hat{Y} = B_0 + B_1Z_1 + B_2Z_2 \dots \dots B_nZ_n$$

If  $n = 2$  and  $Z_2 = Z_1^2$ ,

$$\hat{Y} = B_0 + B_1Z_1 + B_2Z_1^2$$

or simply,  $\hat{Y} = B_0 + B_1Z + B_2Z^2$  Eq 17

Where:

$B_0, B_1, B_2 \dots =$  coefficients

$\hat{Y} =$  dependent variable

$Z_1 =$  first independent variable

$Z_2 =$  second independent variable

.

$Z_n = n^{\text{th}}$  independent variable

Equation 17 is a quadratic equation which is often used for curvilinear fitting. Since the plot for Performance Curve I and II on log-log paper is curvilinear, a second order model has been investigated.

To fit the data for  $U/L$  vs  $G/L^{1.2}$  in this study,

$$\hat{Y} = \log U/L$$

$$Z = \log G/L^{1.2}$$

In most regression problems, the postulated mathematical model is linear in the unknown coefficients. Methods of

solving sets of simultaneous linear equations, whose coefficients form a symmetrical matrix can be found in many statistics books. One of these methods which has been often used is known as the Abbreviated Doolittle Method (Ostle 1964). This method is well suited to programming for digital computers as well as being useful when only desk calculators are available. For convenience in this study, a computer program was prepared by the Iowa State University Computer Center to make the multiple regression analyses. The coefficients in the regression equation thus calculated for Performance Curve I are as follows:

$$B_0 = -0.208$$

$$B_1 = +1.950$$

$$B_2 = -0.645$$

Thus, the curve can be expressed mathematically as follows:

$$\begin{aligned} \log U/L = & -0.208 + 1.950 \log G/L^{1.2} \\ & -0.645 (\log G/L^{1.2})^2 \end{aligned} \quad \text{Eq 18}$$

## 2. Performance Curve II

Similarly, by combining the data from a curve of  $U/L$  vs  $G/L^{1.2}$  with similar data from a curve of  $U/L$  vs  $R/L^{1.6}$ , one can get values of  $R/L^{1.6}$  corresponding to values of  $G/L^{1.2}$ . A multiple regression analysis of these data can be used to define the relation between  $R/L^{1.6}$  and  $G/L^{1.2}$ . In this analysis,

$$\hat{Y} = \log R/L^{1.6}$$



$$Z = \log G/L^{1.2}$$

The resulting coefficients from the regression analysis are:

$$B_0 = -3.250$$

$$B_1 = +1.013$$

$$B_2 = -0.036$$

$$\begin{aligned} \text{Then, } \log R/L^{1.6} = & -3.250 + 1.013 \log G/L^{1.2} \\ & -0.036 (\log G/L^{1.2})^2 \end{aligned} \quad \text{Eq 19}$$

Where G and R have been defined on page 108.

### 3. Uses of performance curves

These two curves as shown in Figures 41 and 42 depict the filter performance for the given type of influent suspension through the given type of granular media.

a. As long as the type of influent suspension and filter media remain constant the filter performance can be predicted satisfactorily using such curves for various selections of influent concentration, grain size, flow rate, depth of bed, length of filter run, effluent quality and head-loss. For the variables of influent concentration, flow rate, and grain size, the prediction should preferably not be attempted beyond the limits of the pilot scale investigation. By the use of such performance curves, an existing plant can be reviewed for possible improvement, or a new plant can be designed rationally.

b. Where the water source and the treatment practice are quite similar in a region of the country, such as iron removal

Figure 41. Performance Curve I:  $U/L$  vs  $G/L^{1.2}$ ,  
lumped data for various grain sizes,  
flow rates and influent concentrations

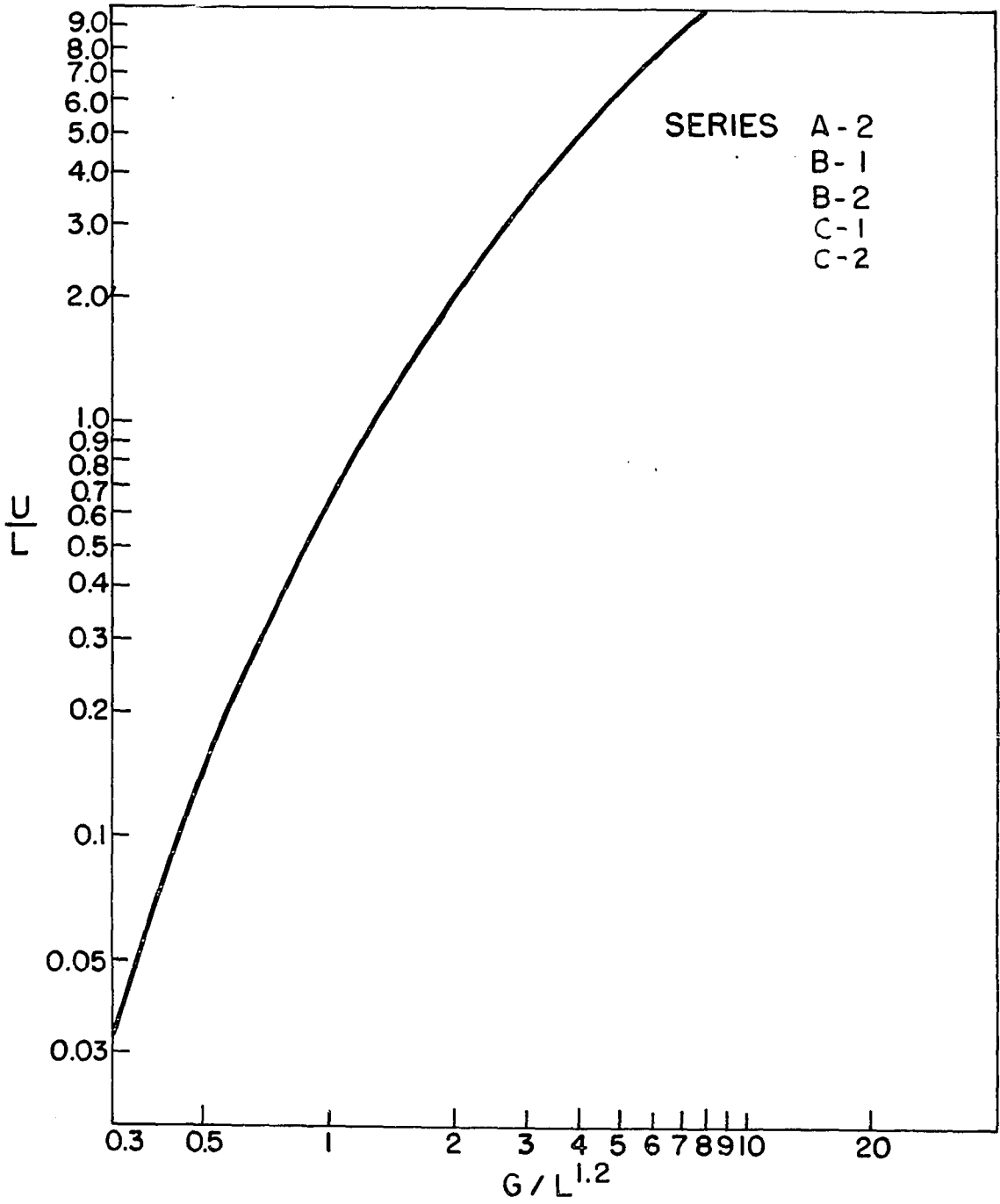
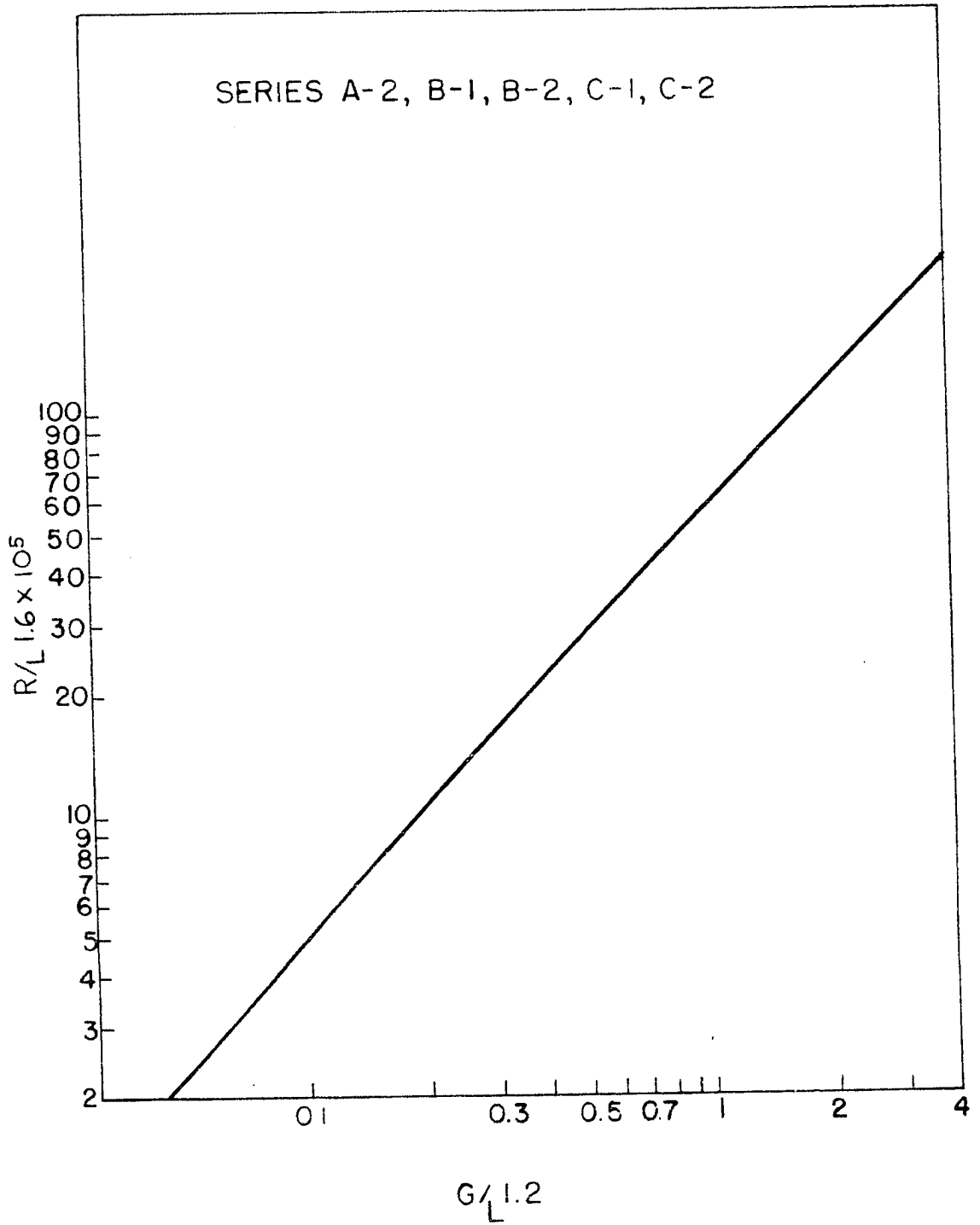


Figure 42. Performance Curve II:  $R/L^{1.6}$  vs  $G/L^{1.2}$ ,  
lumped data for various grain sizes,  
flow rates and influent concentrations



or lime softening of well waters, the performance curves may be found quite similar. Thus, less effort may be required to define the performance curves.

c. If the type of media is standardized in pilot filtration experiments with an apparatus similar to that used in this study, the performance curves will describe the filtering characteristics of the influent suspension. Thus, a filterability index may be derived or at least various types of influent suspensions may be classified into several broad categories for design purposes.

#### E. Criteria for Evaluation of Filtration Variables

Two different criteria should be distinguished in evaluating the effects of filtration variables on filter performance. These have frequently been overlooked. One is based on effluent quality and the other is based on terminal head-loss increase.

In an existing plant where the depth of sand and available terminal head-loss are fixed, the filtration rate, grain size, and filter run length can be selected, so the effluent turbidity just reaches its acceptable limit when the full terminal head-loss occurs, and so minimum production costs are achieved. In designing a new plant, however, many combinations of all of the five variables mentioned above could be selected to achieve an optimum design.

### 1. Effluent quality criterion

From Figure 32, it has been noted that for a given value of  $U$ , the grouped term  $G$  or  $Q^{0.29}d^{0.62}t$  was equal to a constant for all conditions of this study. For a permissible effluent quality at the end of a filter run,  $U$  and  $t$  are fixed for any given depth; thus, the selection of flow rate and grain size will be mutually dependent of each other. In other words, for a given type of suspension, if the effluent quality, depth and grain size are fixed, the flow rate will also be fixed automatically. Any flow rate greater than this rate would produce a poorer filtrate than desired. In order to meet the same filtrate quality criterion, an increase of flow rate must be accompanied by a decrease of grain size. Under the conditions of this study, the flow rate must vary inversely with approximately the 2.1 power of the grain size. This is similar to that reported by Stanley (1955), who stated that for the same "penetration",  $Q$  varies inversely with about the 1.6 power of  $d$ .

### 2. Head-loss criterion

Figure 42 shows that the plot for  $R/L^{1.6}$  vs  $G/L^{1.2}$  has an approximate slope of 0.95 for the lower portion of the curve. This portion of the curve ( $G/L^{1.2}$  approximately less than 1.0) would apply for the typical grain size, flow rate and depth of bed in practical use today. Considering the linear relationship on log-log paper the following expression can be obtained:

$$R/L^{1.6} \propto (G/L^{1.2})^{0.95}$$

or

$$\frac{d^{2.5}(H_t-H_o)}{Q^{1.2}C_o^{1.4}L^{1.6}} \propto \left[ \frac{d^{0.62}Q^{0.29}t}{L^{1.2}} \right]^{0.95}$$

Approximately,

$$(H_t-H_o) \propto Q^{1.5}C_o^{1.4}L^{0.5}t/d^{1.9} \quad \text{Eq 20}$$

The similarity of equations 20 and 8 (p. 16) is apparent.

When  $(H_t-H_o)$ ,  $t$ ,  $L$  and  $C_o$  are predetermined, the ratio  $Q^{1.5}/d^{1.9}$  will also be fixed. This head-loss criterion implies that under the given conditions, an increase of flow rate must be accompanied with an increase of grain size, so that the same increase of head-loss can be reached in a given filtration time. Under the condition of this study, the flow rate must vary directly with approximately the  $1.9/1.5 = 1.3$  power of the grain size based on this head-loss criterion. As expected, this relationship is in the opposite direction of the effluent quality criterion described on the previous page. Thus, an increase of grain size will be favorable from the standpoint of head-loss, but it will produce a poorer effluent if other filtration conditions remain the same. An increase of flow rate will not favor either the effluent quality or head-loss. When the flow rate and grain size are both increased, the value of  $G$  is increased and consequently a higher value of  $U$  will result at the same  $t$ , which indicates a worse effluent.

If the head-loss criterion is used, the filter run will



be affected by other variables as follows:

$$t \propto d^{1.9},$$

$$t \propto 1/Q^{1.5}$$

These results are consistent with those reported by Baylis (1926), Ling (1955), and Dostle and Robeck (1966).

#### F. Minor Observations

##### 1. Comparison between uniform and graded sand filters

The experimental runs in series D were conducted for comparison of uniform and graded sand filters. The results were similar to those reported by Ling (1955) as mentioned on page 21. The graded sand filter with an effective size of 0.435 mm and uniformity coefficient of 1.38 required a deeper bed than the uniform sand filter of 0.386 mm grain size to produce the same effluent quality. However, the head-loss for the uniform sand filter was significantly higher. Table 4 presents the results from a typical run, No. 57. The explanation will be presented in next sub-section.

##### 2. Iron removal is essentially a surface phenomenon

The following observations reveal that the iron removal process in sand filters is essentially a surface phenomenon.

a. Typical differences between uniform and graded sand filters have been presented above. The explanation may be largely due to the dominant removal mechanism. Since the coarser grains have less total surface area but higher permeability than the fine grains, the additional volume of

Table 4. Comparison of filters producing filtrates of equal quality

Run No. 57	Uniform sand filter	Graded sand filter
Q (gpm/sq ft)	6.0	6.0
$\bar{C}_O$ (mg/l)	3.46	3.46
d (mm)	0.386	-
effective size (mm)	-	0.435
unif. coef.	-	1.38
t (hr)	6.5	6.5
L (in.)	6.0	8.0
$H_t - H_o$ (ft)	3.39	1.66
$C/C_o$	0.04	0.04

media required in the graded sand filter to achieve equal effluent quality leads to two-fold results: (1) it makes up the difference in grain surface area so that the coarser grains can do equal removal because sufficient grain surface area has been provided; (2) it increases the volume of voids and consequently decreases the expected head-loss because the increase of head-loss is mainly a function of the volume of unclogged voids as suggested by Camp (1964) and Ives (1967). While the total suspended particles retained in both filters are about the same because of approximately equal removal efficiencies, the total un-clogged volume would be larger,

and consequently, less head-loss would be expected for the graded sand filter under discussion.

b. The observed data for the removal efficiency as reflected in Figure 32 are less scattered than that for the head-loss data as reflected in Figure 36. The differences in scatter noted above indicate that the slight variations in the compactness of filter bed will result in less significant changes in removal efficiency than in head-loss. Some variation in the compactness of the filter bed after backwashing was expected. The granular media in a plant scale filter will be loosely packed in an inconsistent manner after backwashing from filter run to filter run. A consistent degree of compactness of filter bed can not be expected (Ives and Sholji 1965). Therefore, in the pilot plant experiments, it was not intended to control rigorously the compactness of the filter bed, so that the actual condition in a plant scale filter bed could be better simulated. Furthermore, an overly compacted bed would yield better effluent quality and higher head-losses than expected for a full scale, loosely packed bed. If a procedure were used to obtain uniform and a higher degree of compactness, the prediction results may be in error and the factor of safety in the design would tend to be reduced.

The differences in scatter of the data also suggest that the iron removal mechanism is not significantly affected by the slight variation in bed compactness which would imply that interstitial straining was not a dominant mechanism for

this suspension.

### 3. Catalytic effect of iron deposits on the removal efficiency

It was noted in runs 26 and 27 that a previously unused clean sand filter had slightly lower removal efficiency than a sand used in one filter run, which probably had a light coating of ferric oxide deposits which had not been backwashed out. However, such catalytic effect was rather insignificant. As soon as the coating became heavier over a period of several runs, the removal efficiency decreased steadily, probably due to the change of the grain surface character. The sands at the top layer were more liable to be coated and could be re-distributed through the full depth after backwashing. The percentage of coated grains in the whole filter bed would be less for the deeper filters. But for a very shallow filter, perhaps all grains were coated. Therefore, the re-distribution of grains within the bed became meaningless. The 1 in. filter used in this study became noticeably colored after about five runs. Table 5 shows typical results of the experimental runs illustrating the effects of the iron coating.

Table 5. Comparison of removal efficiency due to iron coating effects

Run No.	Filtration conditions	t hr	c/c <sub>o</sub> *			
			Sand A	Sand B	Sand C	Sand D
26	Q = 6 gpm/sq ft d = 0.649 mm $\bar{c}_o = 3.3$ mg/l	1.0	0.81	-	-	-
		1.5	0.79	-	-	-
		4.0	0.81	-	-	-
		6.0	0.75	-	-	-
		7.0	0.73	-	-	-
27	Q = 6 gpm/sq ft d = 0.649 mm $\bar{c}_o = 3.3$ mg/l	1.0	-	0.73	-	-
		3.0	-	0.74	-	-
		5.1	-	0.82	-	-
		7.0	-	0.75	-	-
31	Q = 3 gpm/sq ft d = 0.649 mm $\bar{c}_o = 3.3$ mg/l	1.0	-	-	-	0.58
		1.5	-	-	-	0.59
		2.0	-	-	-	0.64
		4.8	-	-	-	0.77
		6.3	-	-	-	0.79
		10.7	-	-	-	0.79
42	Q = 3 gpm/sq ft d = 0.649 mm $\bar{c}_o = 3.3$ mg/l	1.0	-	0.59	0.65	-
		1.5	-	0.62	0.67	-
		2.0	-	0.64	0.69	-
		3.6	-	0.68	0.73	-
		5.5	-	0.77	0.79	-
		9.0	-	0.80	0.83	-
		9.5	-	0.78	0.83	-

\* Sand A had not been used before.  
Sand B had been used in one previous run.  
Sand C had been used for 5 runs after acid washing.  
Sand D had been used for 5 runs without acid washing.

## IX. METHOD OF PREDICTION OF FILTER PERFORMANCE

Experimental data obtained from three thin layer filters, can be used to predict the filter performance for a deeper filter bed and for longer runs than were included in the pilot scale experiments. Either an extrapolation method or the performance curves can be used for prediction of filtrate quality and increase of head-loss.

## A. Graphical Extrapolation Method

1. For a single grain size, flow rate and influent concentration

a. The U vs t and equi-U plots are prepared as described on pages 63 and 64, and shown in Figures 10 and 11.

b. Extend the equi-U curves to the desired prediction depth beyond the experimental range, such as at points a, b, and c, (Figure 11) for L = 20 in. and find the respective values of t.

c. Plot a, b, and c in Figure 10. The curve will represent the removal efficiency of a 20 in. filter, for which the effluent quality can be predicted by reversing the procedure described on page 63.

d. Prepare U vs ( $H_t - H_0$ ) and respective equi-U plots as shown in Figure 13 and 14.

e. Find e, f, and g by extrapolation on Figure 14, and plot in Figure 13 in a similar way. The curve will represent the increase of head-loss of a 20 in. filter. The head-loss

and effluent quality for a given time can be obtained by using the predicted curves in Figures 10 and 13 for any common value of  $U$ .

2. For various grain sizes flow rates and influent concentration

Prediction can also be made for filtration through media of different grain size and at different flow rates from those used in pilot experiments. The same procedures are followed as described in previous sub-section except that  $U$  vs  $G$ ,  $U$  vs  $R$  and respective equi- $U$  plots as shown in Figures 32, 35, 36 and 39 would be used instead.

B. Prediction by Use of Performance Curves

Performance curves for the given suspension such as shown in Figures 41 and 42 must first be prepared. The prediction of filter performance for other grain sizes, flow rates, or influent concentrations could then be made more easily than by graphical extrapolation.

1. For uniform grains

When  $d$ ,  $Q$ ,  $L$  and  $t$  are given,  $G/L^{1.2}$  can be calculated from equation 15, and  $U/L$  can be found from Performance Curve I as shown in Figure 41 (or by equation 18 on p. 138). With  $U$  and  $t$  known  $C/C_0$  can be predicted from the Table 10 or Figure 50 or 51 in the Appendix.

$R/L^{1.6}$  can be found from the Performance Curve II as shown in Figure 42 (or by equation 19 on p. 139) for the value of

$G/L^{1.2}$  just obtained above. The increase of head-loss can be found from equation 16.

Check runs using filter Apparatus B for the applicability of the proposed method for prediction have been made. The typical results are listed in Table 6, using uniform sands of the same size in both Apparatus A and B. It can be noted that in Table 6 the observed increase of head-loss in Apparatus B, which has a deeper bed, is always lower than the predicted value using Apparatus A by an amount of about 0.5 to 1.0 ft. The discrepancy is believed to be due to the different arrangement of these two apparatus. Apparatus A was made of light weight stainless steel. The filter wall extended above the sand surface only about an inch as used in series A and B. An extension unit had to be added on for backwashing. After backwashing, the extension unit was removed and the filter top was replaced before starting filtration. During these operations, certain jarring action on the filters could not be avoided and would result a more compacted beds in Apparatus A than in Apparatus B, for which no such procedure was involved. However, the discrepancy is on the safe side and may be reduced if the construction of the thin filters is improved.

## 2. For graded grains

When uniform sand is used in pilot plant experiment, the information may be applied to the prediction of performance of a graded sand filter. However, an equivalent uniform grain size, as defined below, must be used in place of  $d$ , the



uniform size, which had been used in developing the performance curves.

Table 6. Prediction results of typical check runs at specific times

Run No.	d mm	Q gpm/sq ft	C <sub>o</sub> mg/l	L in.	t hr	$\frac{G}{L^{1.2}}$
18	0.545	3.0	3.40	21.5	13.8	0.328
20	0.545	3.0	3.30	21.5	7.0	0.166
37	0.649	4.5	5.92	9.0	5.5	0.466
39	0.649	6.0	5.70	19.5	9.0	0.329
40	0.649	3.0	5.70	19.5	11.0	0.328
55	0.386	3.0	3.00	5.33	12.0	1.230
57	0.386	6.0	3.46	6.0	6.5	0.704

Run No.	$\frac{U}{L}$	U	$\frac{R 10^5}{L^{1.6}}$	C/C <sub>o</sub>		(H <sub>t</sub> -H <sub>o</sub> ), ft	
				pred.	obs.	pred.	obs.
18	0.042	1.67	17.8	< 0.01	< 0.01	2.26	1.50
20	0.063	0.06	8.5	< 0.01	< 0.01	1.04	0.45
37	0.137	1.24	26.0	0.03	0.03	1.89	2.41
39	0.044	0.86	17.9	< 0.01	< 0.01	5.95	4.69
40	0.044	0.86	17.8	< 0.01	< 0.01	2.57	2.23
55	0.916	4.88	65.0	0.035	0.023	1.78	2.00
57	0.310	1.86	40.0	0.04	0.04	3.64	3.39

The upper layer of a graded sand filter will do most of the removal during a typical length of filter run. Therefore, an equivalent uniform grain size,  $d_{eq}$ , which represents the average grain size of upper part of the bed, may be used for prediction of filter performance:

$$\begin{aligned} d_{eq} &= 1/2 (P_{10} + P_{60}) \\ &= 1/2 \text{ effective size } (1 + \text{unif. coef.}) \end{aligned}$$

which is about the size of  $P_{30}$ .

Where:

$P_{10}$ ,  $P_{30}$ ,  $P_{50}$ ,  $P_{60}$  represent the size of grain in mm such that 10%, 30%, 50%, 60% respectively of the particles by weight are smaller than the stated size.

$$\text{Effective size} = P_{10}$$

$$\text{Unif. coef.} = P_{60}/P_{10}$$

For long filter runs, a larger portion of the filter bed will aid in significant removal. The representative equivalent grain size will be even coarser, and may be calculated from the following expression:

$$d'_{eq} = \frac{\sum P_i d_i}{100}$$

Where:

$p_i$  = weight fraction of sample (%) separated between adjacent sieves

$d_i$  = geometric mean size of adjacent sieve openings

$d'_{eq}$  is about the size of  $P_{50}$ . For simplicity and higher factor of safety,  $P_{60}$  may be used as the equivalent uniform grain size, which is equal to the effective size times the

uniformity coefficient.

The following paragraph will describe the method for prediction of the performance for a graded sand filter.

a. Operation conditions for Run 57.

Graded sand size	$P_{10} = 0.435$ , effective size
	$P_{60}/P_{10} = 1.38$ , unif. coef.
Depth of bed	$L = 8''$
Flow rate	$Q = 6.0$ gpm/sq ft
Length of filter run	$t = 6.5$ hr
Influent concentration	$C_o = 3.46$ mg/l iron

b. Prediction of effluent quality and head-loss at 6.5 hr after starting the filtration.

Case 1.

$$\text{Use } d = d_{eq} = 1/2 (P_{10} + P_{60})$$

$$d = 1/2 (0.435 + 0.60) = 0.518 \text{ mm}$$

$$G = (6)^{0.29} (0.518)^{0.62} (6.5)$$

$$= 7.27$$

p. 108

$$R = (0.518)^{2.5} (H_t - H_o) / (6)^{1.2} (3.46)^{1.4}$$

$$= 3.96(H_t - H_o) \times 10^{-3}$$

p. 108

$$G/L^{1.2} = 7.27 / (8)^{1.2} = 0.60$$

$$U/L = 0.215$$

Fig. 41

$$U = 1.72$$

$$C/C_o = 0.04$$

Appendix  
Fig. 50, 51

$$R/L^{1.6} = 34 \times 10^{-5}$$

Fig. 42

$$R = (8)^{1.6} \times 34 \times 10^{-5} = 3.96(H_t - H_o) \times 10^{-3}$$

$$(H_t - H_o) = 2.39 \text{ ft}$$

Case 2.

$$\text{Use } d = d'_{eq} = \sum p_i d_i / 100$$

$$d'_{eq} = 0.579 \text{ mm according to the sieve analysis}$$

$$G = (6)^{0.29} (0.579)^{0.62} (6.5) = 7.8$$

$$R = (0.579)^{2.5} (H_t - H_o) / (6)^{1.2} (3.46)^{1.4}$$

$$= 5.25 (H_t - H_o) \times 10^{-3}$$

$$G/L^{1.2} = 7.8 / (8)^{1.2} = 0.645$$

$$U/L = 0.26$$

$$U = 2.08$$

$$C/C_o = 0.06$$

$$R/L^{1.6} = 36 \times 10^{-5}$$

$$H_t - H_o = 1.91 \text{ ft}$$

Case 3.

$$\text{Use } d = P_{60} = 0.6 \text{ mm}$$

$$G = (6)^{0.29} (0.6)^{0.62} (6.5) = 7.97$$

$$R = (0.6)^{2.5} (H_t - H_o) / (6)^{1.2} (3.46)^{1.4}$$

$$= 5.73 (H_t - H_o) \times 10^{-3}$$

$$G/L^{1.2} = 7.97 / (8)^{1.2} = 0.658$$

$$U/L = 0.27$$

$$U = 2.16$$

$$C/C_o = 0.07$$

$$R/L^{1.6} = 37 \times 10^{-5}$$

$$(H_t - H_o) = 1.80 \text{ ft}$$

A comparison of observed and predicted values is shown in Table 7.

Table 7. Comparison of predicted and observed performance results for a graded sand filter

Filtration data	Predicted			Observed
	Case 1	Case 2	Case 3	
$C/C_0$	0.04	0.06	0.07	0.04
$(H_t - H_0)$ , ft	2.39	1.91	1.80	1.66

### C. Applicability of the Method to Other Types of Suspension and Media

Although only one type of the influent suspension has been investigated in this study, available data from other investigators have been analyzed, which include observations made with many different suspensions. The data analyzed were measured from the graphs presented in the published papers cited below. Results indicate that the characteristic curves are similar regardless of the type of suspension.

The proposed method of prediction has been applied to these waters. The predicted results for an arbitrarily selected depth and time are presented in Table 8 on page 176. The various suspensions are discussed below.

Figure 43 shows data from Fox and Cleasby (1966), where the influent suspension was similar to that in this study, but using a uniform sand from Muscatine, Iowa, and at a temperature of 15.6 °C. A good conformity of the observed data with the predicted curve is evident, even though the

experimental conditions are similar but not identical.

Figures 44 and 45 were developed from the data of Ling (1955), where the influent suspension was prepared from Fuller's earth and the tap water of the city of Minneapolis, Minnesota. The raw water was coagulated with freshly prepared solutions of  $\text{FeCl}_3$  and  $\text{Ca}(\text{OH})_2$ . The settled water was used for filtration. A steeper slope for the curves is evident compared to that in this study.

Figures 46 and 47 were developed from the data of Eliassen (1941) at Providence, Rhode Island. The raw water total solids content was about 50 ppm. Ferric sulfate was added as the coagulant and lime was added to bring the pH up to 10.0. The settling period was from 48 to 96 hr. The floc particles going to the filter were very small. The sand had an effective size of 0.46 mm and uniformity coefficient of 1.22. The U-t curves for different depths are spaced closer, which may be attributed to the graded sands and a different water.

Figure 48 was developed from the data of Eliassen et al. (1965) at Stanford, California, where radioactive virus was filtered through soils.

Figure 49 was developed from Ives' data (1961a) where the suspension was prepared from radioactive algae.

It is evident from the shape of the curves, and from the results in Table 8 that the proposed method for prediction of filter performance can be applied equally well to any type

of influent water, filtering through any granular material. Some error might be introduced in the graphical manipulation, but it is insignificant for practical purposes.

Figure 43. U vs G, data from Fox and Cleasby (1966)

$$d = 0.705 \text{ mm}$$

$$Q = 4.0 \text{ gpm/sq ft}$$

$$L = 3.48 \text{ in.}$$

$$\bar{C}_o = 5.70 \text{ mg/l}$$

$$T = 15.6 \text{ }^\circ\text{C}$$

Suspension: hydrous ferric oxide floc



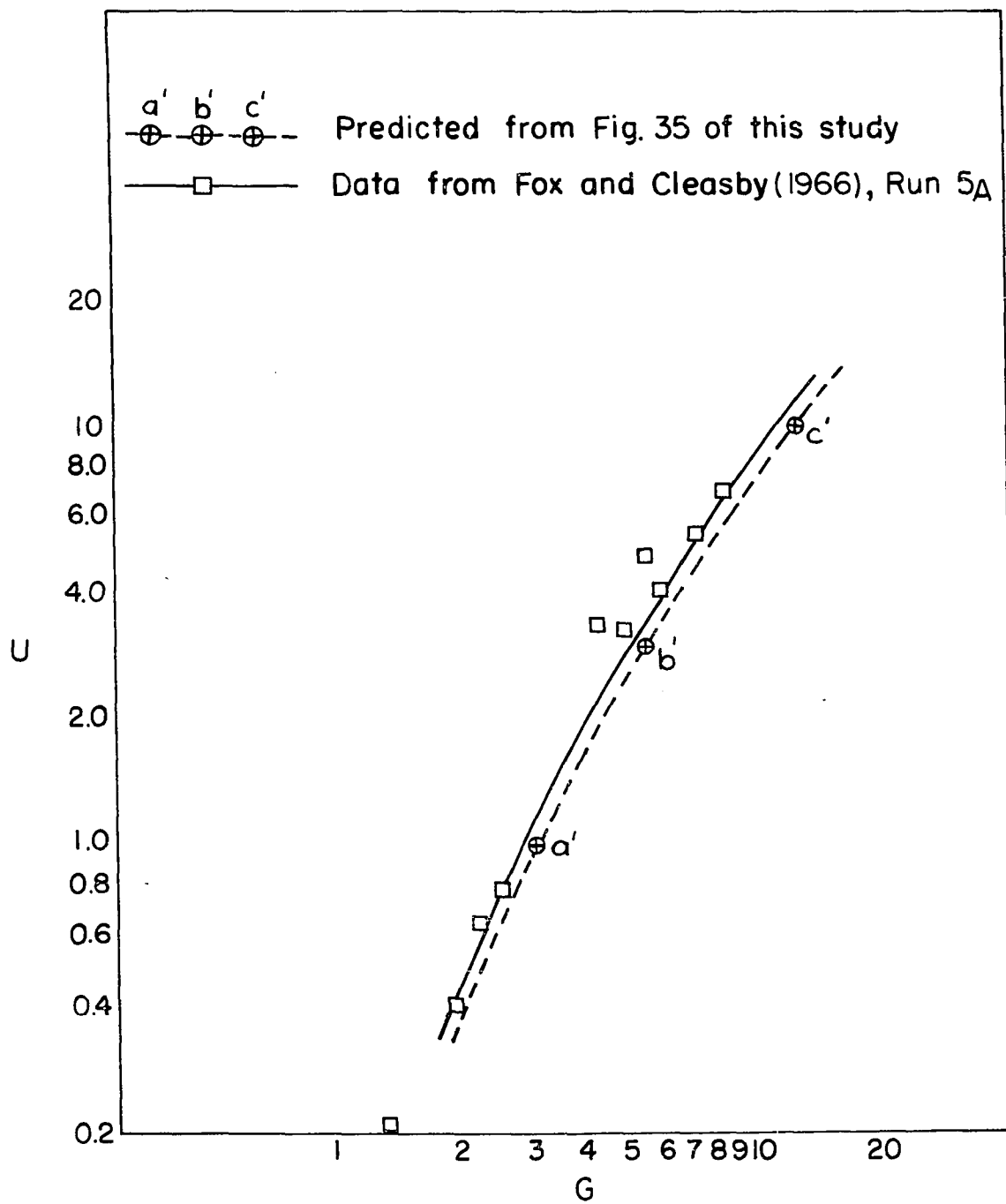


Figure 44. U vs filtration time, data from Ling (1955)

$$d = 0.383 \text{ mm}$$

$$Q = 2.0 \text{ gpm/sq ft}$$

$$\bar{C}_0 = 17.3 \text{ ppm (Turb.)}$$

Suspension: coagulated and settled clay

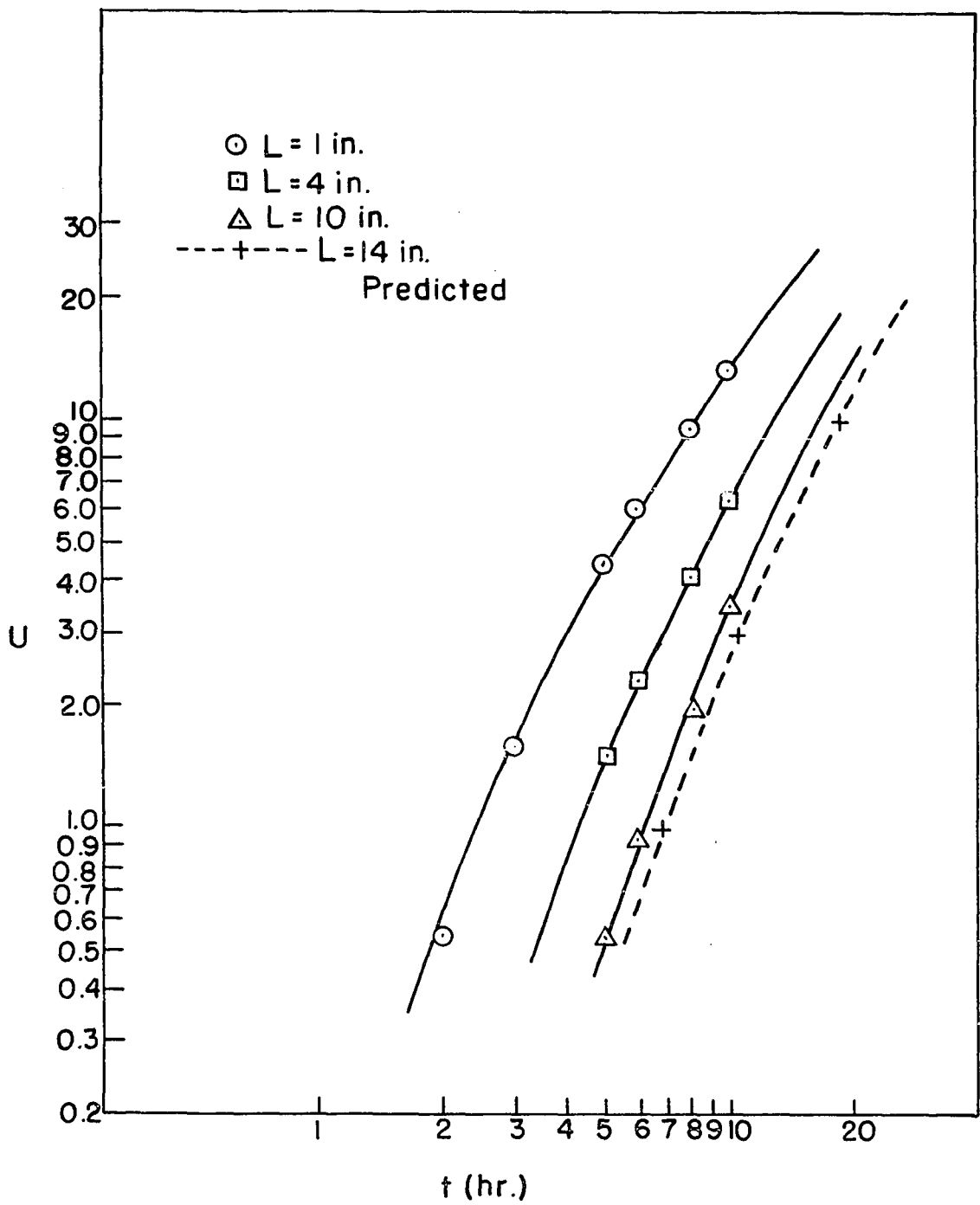


Figure 45. U vs increase of head-loss, data  
from Ling (1955)

$$d = 0.383 \text{ mm}$$

$$Q = 2.0 \text{ gpm/sq ft}$$

$$\bar{C}_o = 17.3 \text{ ppm (Turb.)}$$

Suspension: coagulated and settled clay

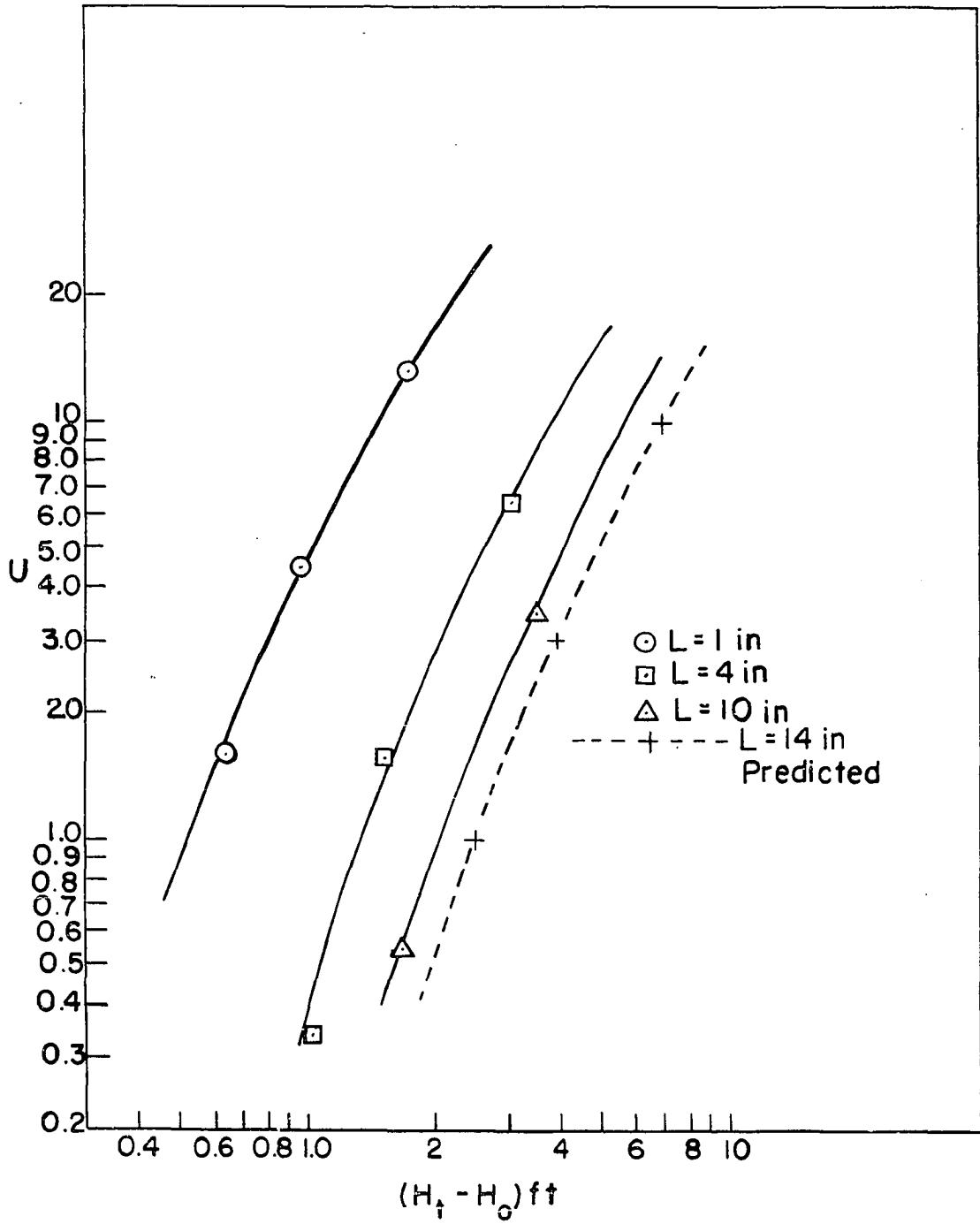


Figure 46. U vs filtration time, data from  
Eliassen (1941)

Effective size = 0.46 mm

Uniformity coefficient = 1.22

Q = 2.0 gpm/sq ft

$\bar{C}_0$  = 0.5 mg/l (iron)

Suspension: coagulated and settled  
water of low solids

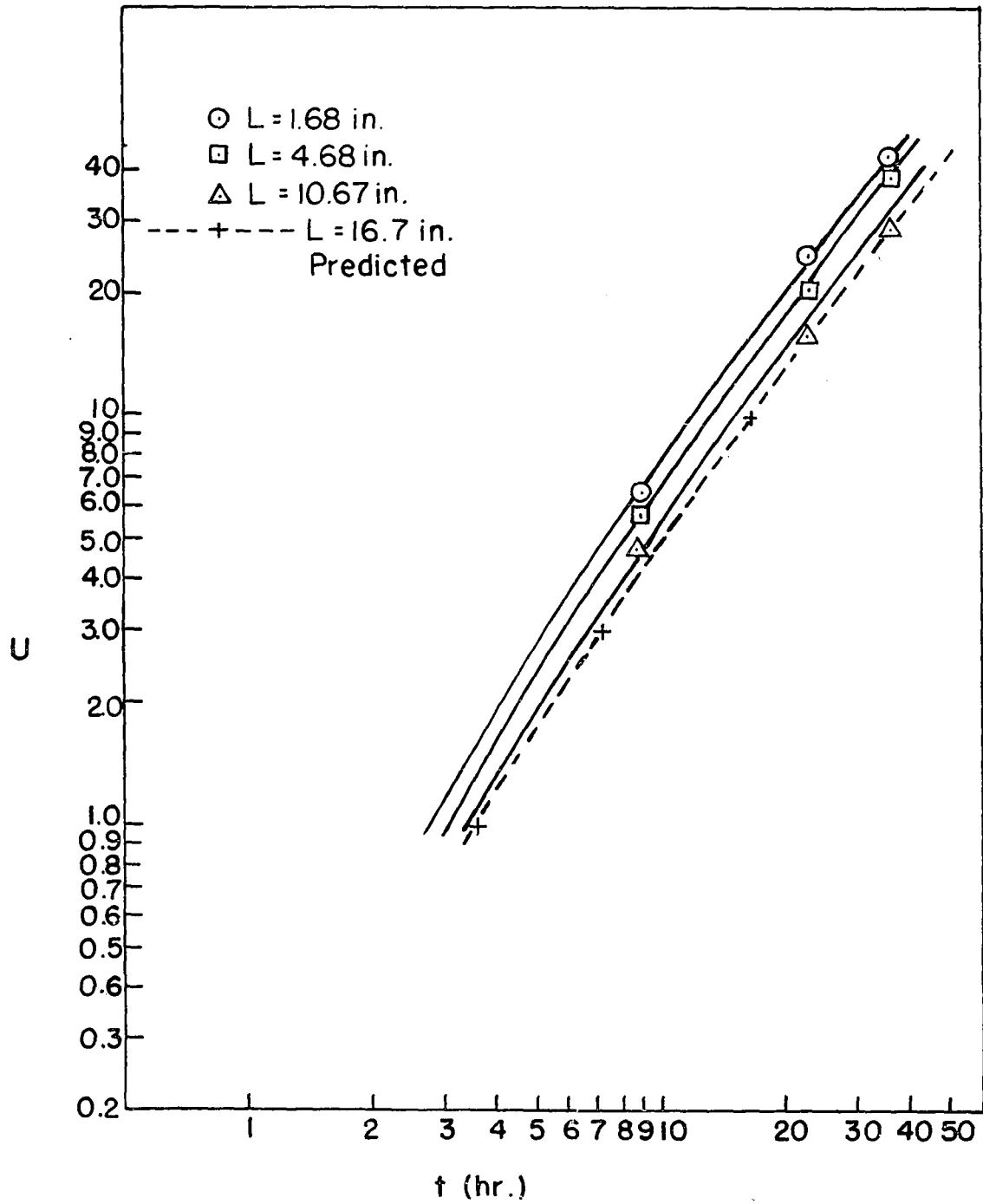


Figure 47. U vs increase of head-loss, data  
from Eliassen (1941)

Effective size = 0.46 mm

Uniformity coefficient = 1.22

$Q = 2.0$  gpm/sq ft

$\bar{C}_o = 0.5$  mg/l (iron)

Suspension: coagulated and settled  
water of low solids



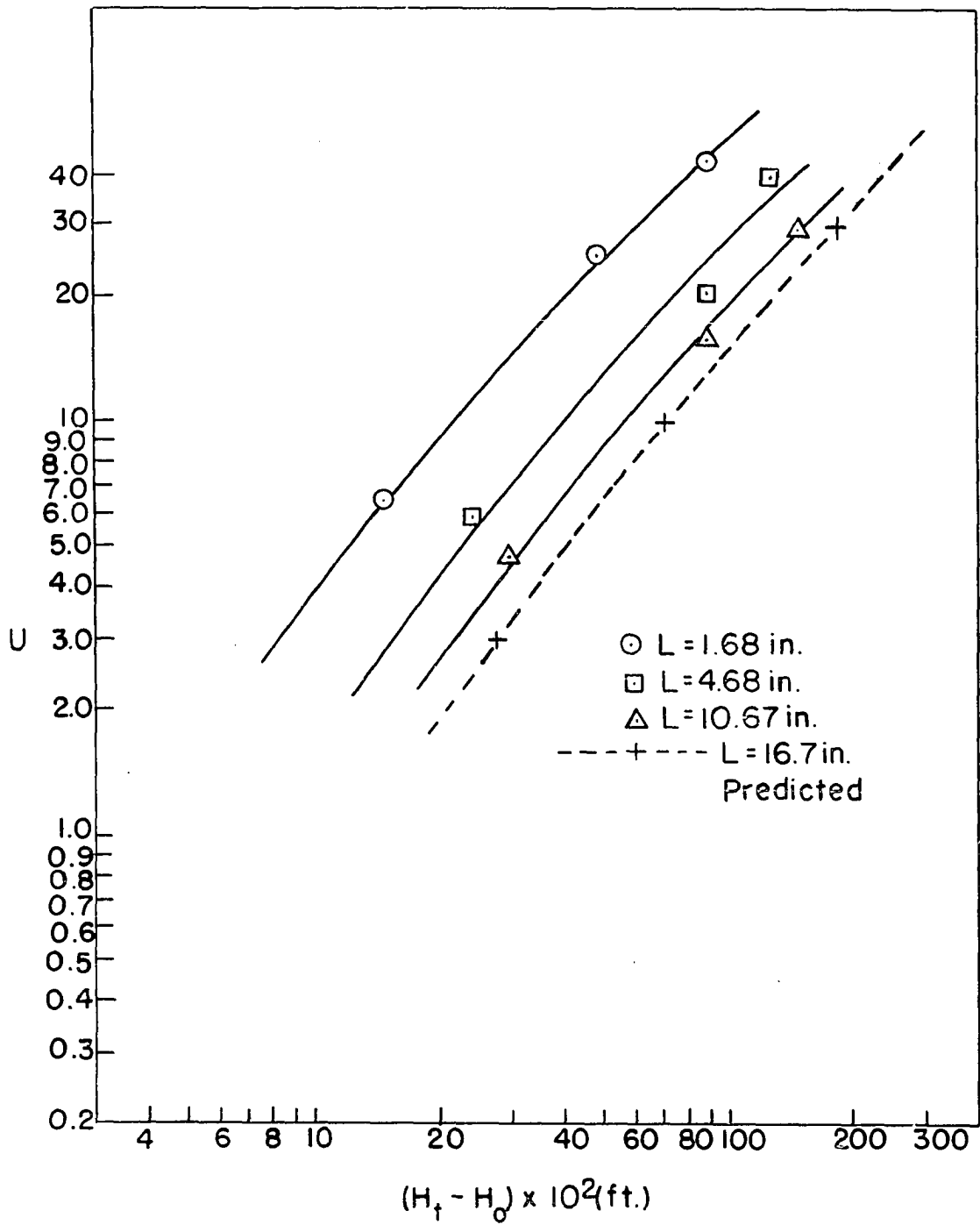


Figure 48.  $U_{100}$  vs filtration time, data from  
Eliassen (1965)

Filter media: soil

$Q = 0.0348$  ml/min/sq cm

$\bar{C}_0 = 6.14 \times 10^3$  cpm/ml (radioactive virus)

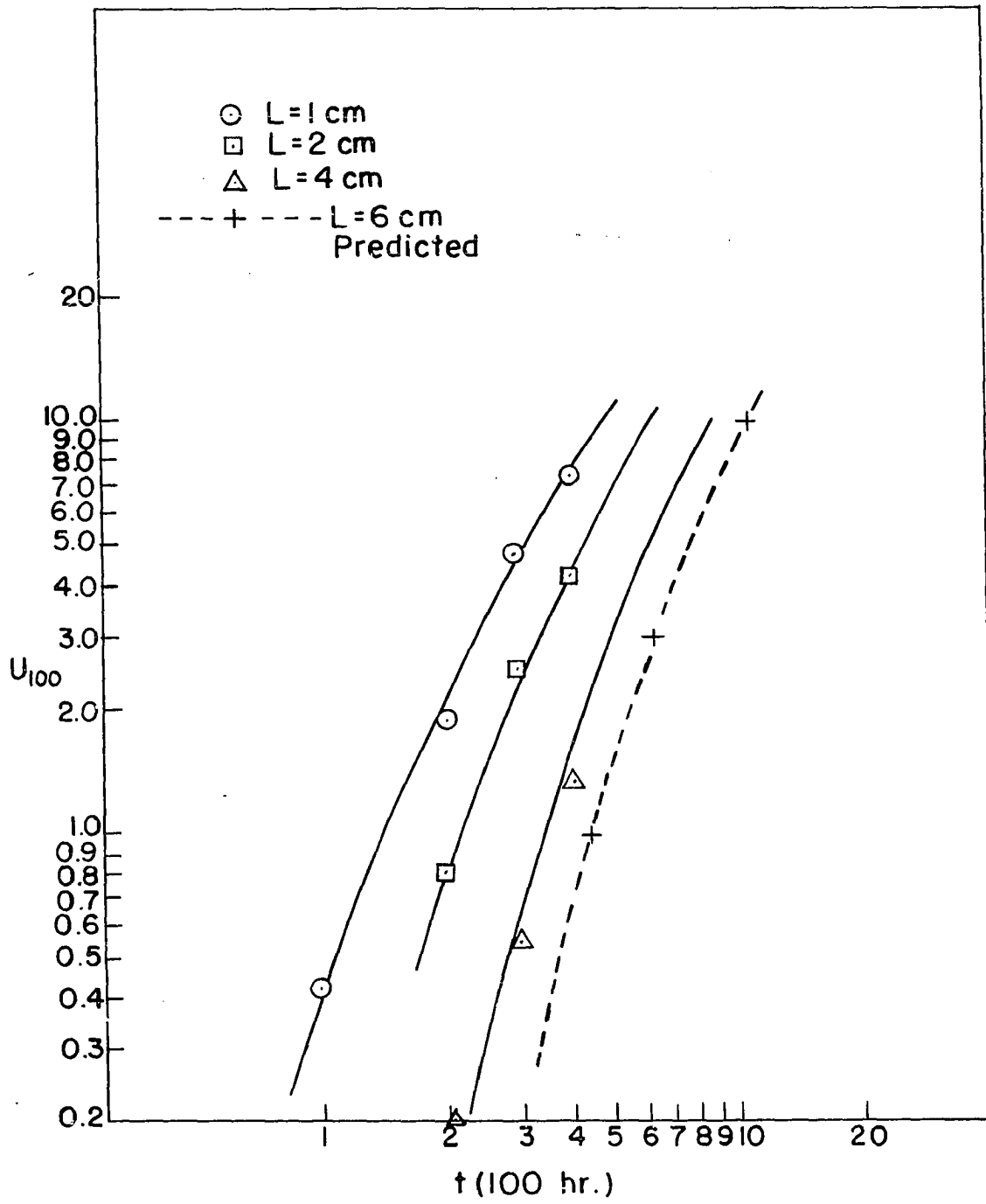


Figure 49. U vs filtration time, data from Ives (1961a)

$$d = 0.544 \text{ mm}$$

$$Q = 2.0 \text{ gpm/sq ft}$$

$$\bar{C}_0 = 135 \text{ ppm (radioactive algae)}$$

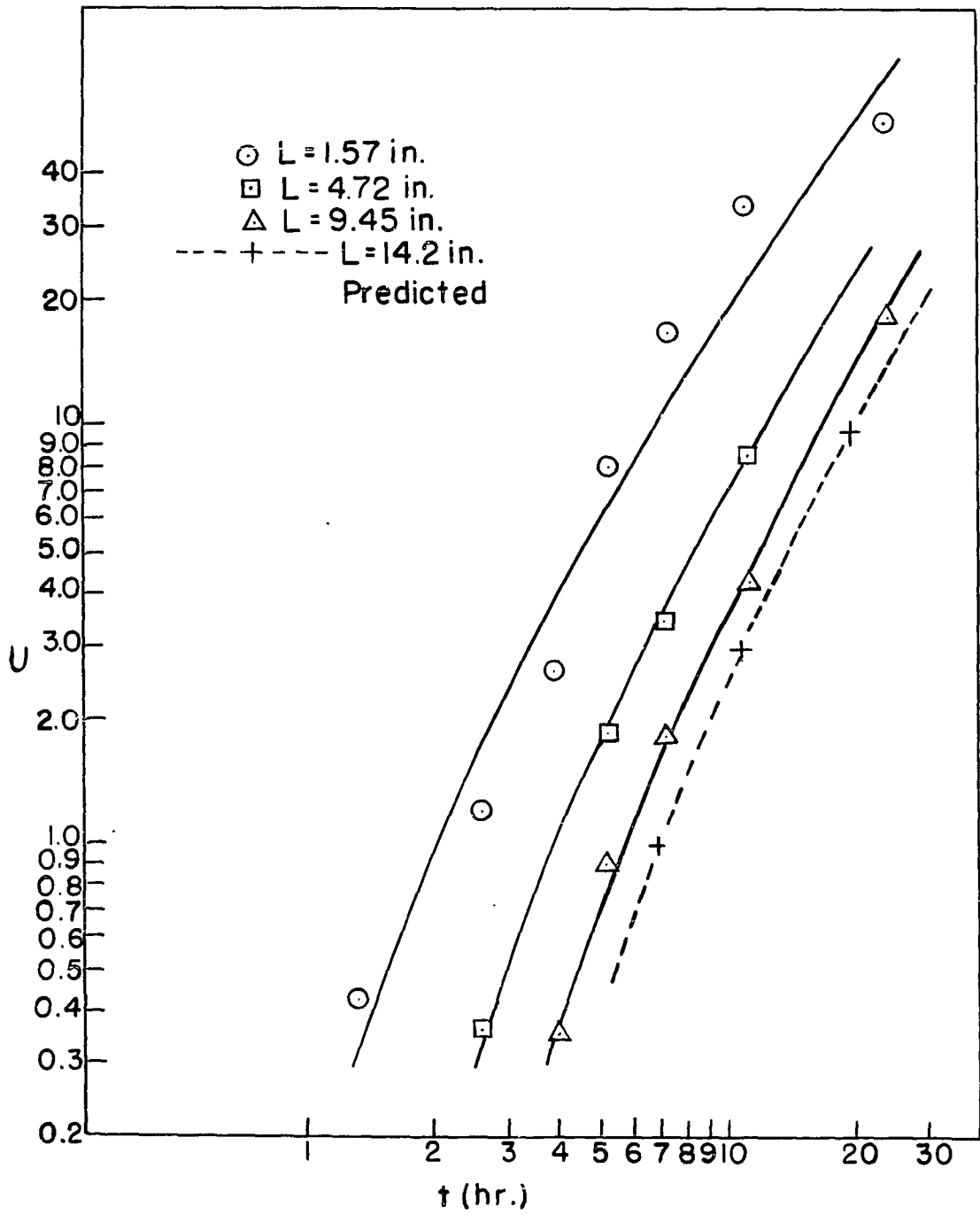


Table 8. Results of prediction of filter performance from the data of other investigators

Source of data	Ling (1955) Run 2	Eliassen (1941) Run 1
Type of influent	Clay susp. coag. with ferric chloride and lime and settled	Raw water of 50 ppm solid, coag. with ferric sulfate and lime and settled
Influent concentration	17.3 ppm (turb.)	0.5 ppm (iron)
Filter media	Uniform sand, Bay City, Wisconsin #40 - #45 d = 0.383 mm	Graded Ottawa sand eff. size = 0.46 mm unif. coef. = 1.22
Flow rate	2.0 gpm/sq ft	2.0 gpm/sq ft
Observation at	L = 1", 4", 10" t = 1 - 10 hr	L = 1.68", 4.68", 10.67" t = 9 - 36 hr
Point for prediction	L = 14" t = 20 hr	L = 16.7" t = 47 hr
Predicted results	U = 11.7 C/C <sub>0</sub> = 0.07 C = 1.2 ppm H <sub>t</sub> -H <sub>0</sub> = 7.8 ft	U = 41 C/C <sub>0</sub> = 0.27 C = 0.14 ppm H <sub>t</sub> -H <sub>0</sub> = 2.4 ft
Obs. values	C = 1.1 ppm H <sub>t</sub> -H <sub>0</sub> = 7.2 ft	C = 0.12 ppm H <sub>t</sub> -H <sub>0</sub> = 2.0 ft

Table 8. (Continued)

Source of data	Eliassen (1965) Fig. 17, p. 31	Ives (1961a) Fig. 9, p. 33
Type of influent	Radioactive T <sub>2</sub> virus	Radioactive algae
Influent concentration	6.14 x 10 <sup>3</sup> cpm/ml	135 ppm
Filter media	Soil No. 3 sp gr 2.71 bulk density 1.17 g/cm <sup>3</sup> porosity 57%	Uniform sand 0.544 mm
Flow rate	0.0348 ml/min/sq cm	2 gpm/sq ft
Observation at	L = 1 cm, 2 cm, 4 cm t = 100 - 400 hr	L = 1.57", 4.72", 9.45" t = 1.33 - 24 hr
Point for prediction	L = 6 cm t = 600 hr	L = 14.2 in t = 24 hr
Predicted results	U = 2.5 C/C <sub>0</sub> = 1.3	U = 14.2 C/C <sub>0</sub> = 0.05
Obs. values	C/C <sub>0</sub> = 0.9 Head-loss data are not available	C/C <sub>0</sub> = 0.02 Head-loss data are not available

## X. PROPOSED RATIONAL DESIGN OF RAPID FILTERS

A rational approach to filter design can be achieved after the performance curves have been developed. If the actual influent quality is expected to vary during the year, the prepared curves should be based upon the poorest filtration conditions, in order to provide a safe effluent. The proposed design procedures are illustrated as follows assuming the suspension being filtered had performance curves similar to Figures 41 and 42.

- a. Try a practical length of filter run, say 24 hours.
- b. Set a permissible filtrate quality at the end of the filter run, for example, 0.3 mg/l of iron in iron filtration.
- c. For an influent of 5 mg/l iron the  $C/C_0$  ratio is equal to 0.06, and the corresponding deposit index,  $U = 14.5$ , which is obtained from Table 10 or Figure 50 or 51 in the Appendix.
- d. Try a coarse uniform sand which can be properly backwashed, say  $d = 0.8$  mm.
- e. Try a filter depth, say  $L = 20$  in.
- f. Find  $U/L = 14.5/20 = 0.725$

From Figure 41

$$G/L^{1.2} = 1.07 = d^{0.62} Q^{0.29} t/L^{1.2}$$

$$0.8^{0.62} \times Q^{0.29} \times 24/20^{1.2} = 1.07$$

$$Q = 8.2 \text{ gpm/sq ft}$$



The above calculations indicate that if  $d = 0.8$  mm,  $L = 20$  in. and  $Q = 8.2$  gpm/sq ft, the filter will produce an effluent of 0.3 mg/l of iron at the end of 24 hr filtration.

- g. Check the increase of head-loss expected.

From Figure 42, for  $G/L^{1.2} = 1.07$

$$R/L^{1.6} = 62 \times 10^{-5}$$

$$d^{2.5} (H_t - H_o) / Q^{1.2} C_o^{1.4} L^{1.6} = 62 \times 10^{-5}$$

$$H_t - H_o = 62 \times 10^{-5} \times 8.2^{1.2} \times 5^{1.4} \times 20^{1.6} / 0.8^{2.5}$$

$$= 15.45 \text{ ft}$$

This would be the predicted increase of head-loss at the end of the filter run for the conditions described above.

- h. A certain factor of safety may be applied to take care of uncontrolled filtration conditions by lowering the permissible effluent concentration. For example, if a safety factor of 2 is used, for  $C/C_o = 0.03$ ,  $U = 5.7$  and for same  $d$ ,  $t$  and  $L$ ,

$$U/L = 5.7/20 = 0.285$$

$$G/L^{1.2} = 0.68$$

$$Q = 1.72 \text{ gpm/sq ft}$$

$$H_t - H_o = 1.49 \text{ ft}$$

From a practical standpoint, this combination of coarse sand and very low rate is not a good selection because it results in a very low head-loss.

- i. Many design alternatives can be easily calculated in the same way. Examples of several alternatives are shown in Table 9. By comparison of the cost of many possible alternatives, an optimum selection of the grain size, filtration rate and filter depth can be obtained.
- j. The terminal head-loss can be easily determined by calculation of the initial head-loss from established equations (Fair and Hatch 1933) and adding the increase of head-loss ( $H_t - H_o$ ).
- k. If graded sands are to be used, two design approaches can be used. (1) first select a value of  $d_{eq}$  and find the flow rate and the increase of head-loss as described above; then calculate the effective size of the graded sands according to the definition of  $d_{eq}$  (p. 156), with a selected practical uniformity coefficient; or (2) first select the effective size and uniformity coefficient and calculate the value of  $d_{eq}$  accordingly. For example, if we select  $d_{eq} = 0.8$  mm, and uniformity coefficient = 1.5,  $1/2$  effective size  $(1+1.5) = 0.8$ , from which effective size = 0.64 mm.  
Thus, a graded sand of 0.64 mm effective size and 1.5 uniformity coefficient can be used in place of a uniform sand of 0.8 mm to obtain the same results as shown in Table 9. An example has been presented

on page 157 for predicting the performance of a graded sand filter.

Table 9. Some alternative designs for example with an influent concentration of 5.0 mg/l iron

	d mm	L in.	t hr	Q gpm/sq ft	H <sub>t</sub> -H <sub>o</sub> ft	C/C <sub>o</sub>
A.	0.8	20	24	8.20	15.45	0.06
B.	0.6	10	24	3.55	5.86	0.06
C.	0.8	20	24	1.72	1.49	0.03
D.	0.6	20	24	3.31	6.70	0.03
E.	0.6	22	24	4.05	9.17	0.03
F.	0.8	22	24	2.16	2.11	0.03

## XI. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Much research has been done on the filtration of water through granular material in deep beds. The sanitary engineer would be aided by a simple but reliable method for selecting the suitable grain size, flow rate, depth of bed, filter run length and terminal head-loss for a given influent suspension. Attempts to use rational mathematical analyses for this purpose have been limited in the following respects. The first limitation is that no theoretical formulation can eliminate the need for some empirical constants. The second limitation is that some idealized assumptions must be made as to the nature of the suspension or the filter which may deviate significantly from actual conditions. Furthermore, for a more precise expression, more experimental constants and more complicated operation are needed which reduce the feasibility of use for practical application.

An approach is presented in this study to provide adequate information for filter design. The proposed method for prediction of filter performance is very simple and straight forward. Basic experimental data are obtained from a compact pilot plant unit consisting of 3 thin filters of varying depth. With a slide rule, French curves and some log-log papers, one can do the rest of the work.

The theoretical background of the proposed method is based on the "Random Walk" analogy to the filtration process.

Since the suspended particles in the influent suspension probably have a wide variation in characteristics, such as their size, shape, floc strength etc., a random distribution of such characteristics in the suspension is a natural phenomenon. Furthermore, the media in a granular filter, even if it is uniform in size and character, will be also randomly packed after backwashing. Needless to say, such uniformity of media size can never be expected in practical filtration conditions. In the filtration process, these two random systems interact. How the particles move and where they will be retained in the filter bed can not be traced precisely. However, it may be postulated that any fraction of particles tends to find its own lodging place with more probability than any other fraction. Thus, filtration through granular media is logically a random process and distribution of particles in a filter could be expected to follow some probability law.

Experimental results have shown that the removal of particles per unit depth through the filter bed is quite similar to the chi-square probability distribution which occurs so often in statistical problems that it merits special attention. The variate  $U$  of this distribution can be considered a measure of the clogging process. It has been related to experimental filtration data and tentatively called the deposit index. Performance curves and empirical equations have been developed to describe the filtration for the given suspension through the sand filters used in this study. The

technique has also been applied to other types of suspensions. The method of prediction of filter performance and procedures for rational design of rapid sand filters have been proposed. Examples are presented that show many alternative filter designs may be found for a given suspension to yield desired effluent quality and head-loss. An optimum design may be obtained by comparison of cost of several alternates.

Based on the limited tests conducted in this study, the following conclusions have been reached:

- a. The filtration process may be best described by probability theory because of the random nature of suspension, the filter media, and the transport and attachment mechanisms active in the filter.
- b. The proposed deposit index,  $U$ , which is derived from the chi-square distribution function, is a useful parameter in the study of filter performance. It can be correlated with specific deposit under practical conditions by a simple exponential equation of the form:

$$U = a\sigma^b$$

- c. Two performance curves can be developed for a given type of suspension and filter media. They are sufficient and useful for prediction of filter performance and for rational filter design.
- d. If selection of flow rate and grain size is based on filtrate quality criterion, any increase of flow rate

must be accompanied with a decrease of grain size in order to produce the same effluent quality. Under the experimental conditions of this study, the permissible flow rate is inversely proportional to about the 1.3 power of grain size to achieve the same effluent quality.

- e. If selection of flow rate and grain size is based on head-loss criterion i.e. for same increase of head-loss at any given  $t$ , any increase of flow rate must be accompanied with an increase of grain size. Under the experimental condition of this study, the flow rate is directly proportional to about the square of grain size to achieve the same head-loss in time  $t$ .
- f. A good filter design consists of suitable choice of all the important variables and should not be limited by any arbitrary standards.
- g. The proposed method for prediction of filter performance can be applied to any type of suspension and media. A selected factor of safety can be included in the rational design.
- h. The proposed method is useful in the improvement of existing plants and in design of new filters.

The following recommendations are presented:

- a. Application of the results from this study to filtration of sewage solids may work equally well, and should be investigated.

- b. The temperature effect on the performance curves should be evaluated.
- c. Further investigation on the property of the performance curves may hopefully develop a universal filtrability index to classify different influent suspensions.
- d. Comparison of the similarity of performance curves on a regional basis is recommended wherever the water sources and treatment practice are similar to see if standard average performance curves could be used in design.
- e. It is speculated that even in an integrated design of pretreatment units and filters, it will be possible to use the proposed method of filter performance prediction with varying type of suspensions.



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XIV. APPENDIX



Table 10. Cumulative chi-square distribution \*

v	P <sub>c</sub>						
	0.001	0.01	0.05	0.10	0.20	0.30	0.40
1	0.05157	0.03157	0.02393	0.0158	0.0642	0.148	0.275
2	0.02200	0.0201	0.103	0.211	0.446	0.713	1.02
3	0.0243	0.115	0.352	0.584	1.00	1.42	1.87
4	0.0908	0.297	0.711	1.06	1.65	2.19	2.75
5	0.210	0.554	1.15	1.61	2.34	3.00	3.66
6	0.381	0.872	1.64	2.20	3.07	3.83	4.57
7	0.598	1.24	2.17	2.83	3.82	4.67	5.49
8	0.857	1.65	2.73	3.49	4.59	5.53	6.42
9	1.15	2.09	3.33	4.17	5.38	6.39	7.36
10	1.48	2.56	3.94	4.87	6.18	7.27	8.30
11	1.83	3.05	4.57	5.58	6.99	8.15	9.24
12	2.21	3.57	5.23	6.30	7.81	9.03	10.2
13	2.62	4.11	5.89	7.04	8.63	9.93	11.1
14	3.04	4.66	6.57	7.79	9.47	10.8	12.1
15	3.48	5.23	7.26	8.55	10.3	11.7	13.0
16	3.94	5.81	7.96	9.31	11.2	12.6	14.0
17	4.42	6.41	8.67	10.1	12.0	13.5	14.9
18	4.90	7.01	9.39	10.9	12.9	14.4	15.9
19	5.41	7.63	10.1	11.7	13.7	15.4	16.9
20	5.92	8.26	10.9	12.4	14.6	16.3	17.8
21	6.45	8.90	11.6	13.2	15.4	17.2	18.8
22	6.98	9.54	12.3	14.0	16.3	18.1	19.7
23	7.53	10.2	13.1	14.8	17.2	19.0	20.7
24	8.08	10.9	13.8	15.7	18.1	19.9	21.7
25	8.65	11.5	14.6	16.5	18.9	20.9	22.6
26	9.22	12.2	15.4	17.3	19.8	21.8	23.6
27	9.80	12.9	16.2	18.1	20.7	22.7	24.5
28	10.4	13.6	16.9	18.9	21.6	23.6	25.5
29	11.0	14.3	17.7	19.8	22.5	24.6	26.5
30	11.6	15.0	18.5	20.6	23.4	25.5	27.4

\* Entries in the table are values of U.

Table 10. (Continued)

v	$P_c$							
	0.50	0.60	0.70	0.80	0.90	0.95	0.99	0.999
1	0.455	0.708	1.07	1.64	2.71	3.84	6.63	10.8
2	1.39	1.83	2.41	3.22	4.61	5.99	9.21	13.8
3	2.37	2.95	3.67	4.64	6.25	7.81	11.3	16.3
4	3.36	4.04	4.88	5.99	7.78	9.49	13.3	18.5
5	4.35	5.13	6.06	7.29	9.24	11.1	15.1	20.5
6	5.35	6.21	7.23	8.56	10.6	12.6	16.8	22.5
7	6.35	7.28	8.38	9.80	12.0	14.1	18.5	24.3
8	7.34	8.35	9.52	11.0	13.4	15.5	20.1	26.1
9	8.34	9.41	10.7	12.2	14.7	16.9	21.7	27.9
10	9.34	10.5	11.8	13.4	16.0	18.3	23.2	29.6
11	10.3	11.5	12.9	14.6	17.3	19.7	24.7	31.3
12	11.3	12.6	14.0	15.8	18.5	21.0	26.2	32.9
13	12.3	13.6	15.1	17.0	19.8	22.4	27.7	34.5
14	13.3	14.7	16.2	18.2	21.1	23.7	29.1	36.1
15	14.3	15.7	17.3	19.3	22.3	25.0	30.6	37.7
16	15.3	16.8	18.4	20.5	23.5	26.3	32.0	39.3
17	16.3	17.8	19.5	21.6	24.8	27.6	33.4	40.8
18	17.3	18.9	20.6	22.8	26.0	28.9	34.8	42.3
19	18.3	19.9	21.7	23.9	27.2	30.1	36.2	43.8
20	19.3	21.0	22.8	25.0	28.4	31.4	37.6	45.3
21	20.3	22.0	23.9	26.2	29.6	32.7	38.9	46.8
22	21.3	23.0	24.9	27.3	30.8	33.9	40.3	48.3
23	22.3	24.1	26.0	28.4	32.0	35.2	41.6	49.7
24	23.3	25.1	27.1	29.6	33.2	36.4	43.0	51.2
25	24.3	26.1	28.2	30.7	34.4	37.7	44.3	52.6
26	25.3	27.2	29.2	31.8	35.6	38.9	45.6	54.1
27	26.3	28.2	30.3	32.9	36.7	40.1	47.0	55.5
28	27.3	29.2	31.4	34.0	37.9	41.3	48.3	56.9
29	28.3	30.3	32.5	35.1	39.1	42.6	49.6	58.3
30	29.3	31.3	33.5	36.3	40.3	43.8	50.9	59.7

Table 11. Observed data and derived U values of some typical filter runs

Run No.	t hr	d mm	L in	Q gpm/sq ft	C <sub>o</sub> mg/l	C/C <sub>o</sub>	U	H <sub>t</sub> -H <sub>o</sub> ft
22	1.0	0.545	1.0	6.0	3.40	0.71	1.10	0.15
	2.0	0.545	1.0	6.0	3.22	0.75	2.80	0.29
	3.0	0.545	1.0	6.0	3.26	0.78	4.50	0.42
	5.0	0.545	1.0	6.0	3.26	0.75	6.60	0.77
	7.0	0.545	1.0	6.0	3.14	0.81	10.0	1.08
	9.0	0.545	1.0	6.0	3.36	0.78	12.0	1.39
	1.0	0.545	5.0	6.0	3.40	0.13	0.03	0.23
	2.0	0.545	5.0	6.0	3.22	0.15	0.32	0.50
	3.0	0.545	5.0	6.0	3.26	0.17	0.89	0.78
	5.0	0.545	5.0	6.0	3.26	0.24	2.60	1.44
	7.0	0.545	5.0	6.0	3.14	0.25	4.20	1.95
	9.0	0.545	5.0	6.0	3.36	0.30	6.40	2.60
	1.0	0.545	9.0	6.0	3.40	0.02	0.01	0.26
	2.0	0.545	9.0	6.0	3.22	0.03	0.06	0.55
	3.0	0.545	9.0	6.0	3.26	0.03	0.25	0.85
	5.0	0.545	9.0	6.0	3.26	0.02	0.75	1.65
	7.0	0.545	9.0	6.0	3.14	0.04	2.00	2.31
	9.0	0.545	9.0	6.0	3.36	0.02	2.50	3.05

Table 11. (Continued)

Run No.	t hr	d mm	L in	Q gpm/sq ft	C <sub>o</sub> mg/l	C/C <sub>o</sub>	U	H <sub>t</sub> -H <sub>o</sub> ft	
40	1.0	0.649	1.0	3.0	5.61	0.55	0.56	0.06	
	2.2	0.649	1.0	3.0	5.71	0.67	2.50	0.17	
	2.5	0.649	1.0	3.0	5.71	0.67	2.90	0.22	
	3.0	0.649	1.0	3.0	5.81	0.72	3.90	0.27	
	4.0	0.649	1.0	3.0	5.81	0.68	4.70	0.39	
	7.1	0.649	1.0	3.0	6.04	0.82	10.20	0.64	
	9.5	0.649	1.0	3.0	5.51	0.76	12.50	0.93	
	1.0	0.649	5.0	3.0	5.61	0.12	0.02	0.12	
	2.2	0.649	5.0	3.0	5.71	0.05	0.10	0.30	
	2.5	0.649	5.0	3.0	5.71	0.05	0.25	0.36	
	3.0	0.649	5.0	3.0	5.81	0.06	0.41	0.46	
	4.0	0.649	5.0	3.0	5.81	0.08	0.93	0.66	
	7.1	0.649	5.0	3.0	6.04	0.15	3.40	1.26	
	9.5	0.649	5.0	3.0	5.51	0.29	7.00	1.82	
	1.0	0.649	9.0	3.0	5.61	0.01	0.01	0.15	
	2.2	0.649	9.0	3.0	5.71	0.01	0.01	0.38	
	2.5	0.649	9.0	3.0	5.71	0.01	0.08	0.49	
	3.0	0.649	9.0	3.0	5.81	0.01	0.11	0.62	
	4.0	0.649	9.0	3.0	5.81	0.01	0.30	0.88	
	7.1	0.649	9.0	3.0	6.04	0.03	1.85	1.83	
	9.5	0.649	9.0	3.0	5.51	0.02	3.00	2.65	
	57	1.0	0.386	6.0	6.0	3.41	0.01	0.01	0.32
		1.5	0.386	6.0	6.0	3.46	0.01	0.01	0.62
		2.0	0.386	6.0	6.0	3.46	0.01	0.02	0.87
3.5		0.386	6.0	6.0	3.46	0.01	0.02	1.57	
5.5		0.386	6.0	6.0	3.46	0.02	1.00	2.82	
6.0		0.386	6.0	6.0	3.46	0.03	1.35	3.09	
6.5		0.386	6.0	6.0	3.46	0.04	1.73	3.39	

Table 12. Calculation of specific deposit

Sources of data	Fox and Cleasby (1966), Run 2A					
Filtration conditions	d	= 0.705 mm				
	Q	= 3.86 gpm/sq ft = 15.7 cm/min				
	T	= 15.6 °C				
	L <sub>1</sub>	= 0				
	L <sub>2</sub>	= 3.70 cm				
	δL	= 3.70 cm				
	Q/δL	= 4.25 min <sup>-1</sup>				
	C <sub>0</sub>	= 5.5 mg/l iron				
t (hr)	0	1.00	1.75	2.75	3.25	3.75
δt (min)		60	45	60	30	30
C <sub>1</sub> (mg/l)	5.5	5.5	5.5	5.5	5.5	5.5
C <sub>2</sub> (mg/l)	3.75	3.48	3.73	4.07	4.11	5.30
δC (mg/l)	1.75	2.02	1.77	1.43	1.39	0.20
$\bar{\delta C}$ (mg/l)		1.89	1.90	1.60	1.41	0.80
$\delta \sigma = \bar{\delta C} \delta t Q / \delta L$		482	363	480	180	102
σ (wt/vol)		482	845	1253	1433	1535
σ (vol/vol)*		0.019	0.034	0.050	0.057	0.061
C <sub>2</sub> /C <sub>0</sub>		0.63	0.68	0.74	0.75	0.85
U		0.80	1.95	3.70	4.20	6.40

\* Conversion factor =  $40 \times 10^{-6}$

Table 12. (Continued)

Sources of data	This study, Run 47				
Filtration conditions	d	=	0.649 mm		
	Q	=	4.5 gpm/sq ft = 18.3 cm/min		
	T	=	25 °C		
	L <sub>1</sub>	=	0		
	L <sub>2</sub>	=	2.54 cm		
	δL	=	2.54 cm		
	Q/δL	=	7.23 min <sup>-1</sup>		
	C <sub>0</sub>	=	6.0 mg/l iron		
t (hr)	2.0		2.5	3.0	3.5
δt (min)	120		30	30	30
C <sub>1</sub> (mg/l)	6.00		6.04	6.04	6.04
C <sub>2</sub> (mg/l)	4.88		5.14	5.22	5.40
δC (mg/l)	1.12		0.90	0.82	0.64
$\overline{\delta C}$ (mg/l)	1.12		1.01	0.86	0.73
$\delta\sigma = \overline{\delta C} \delta t Q/\delta L$	970		219	187	158
σ (wt/vol)	970		1189	1376	1534
σ (vol/vol)*	0.039		0.048	0.055	0.061
C <sub>2</sub> /C <sub>0</sub>	0.81		0.85	0.86	0.89
U	2.35		3.10	4.40	5.00

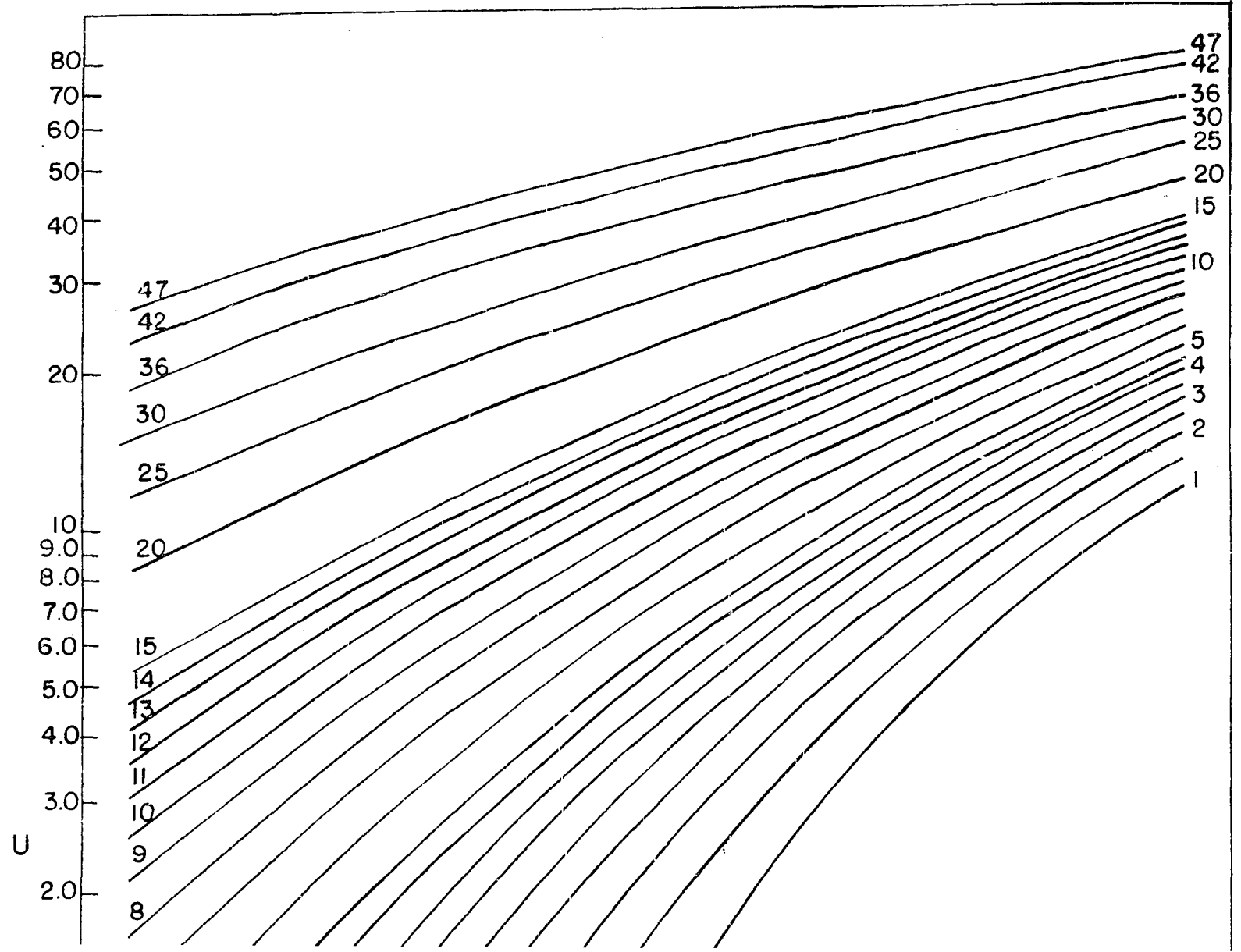
\* Conversion factor =  $40 \times 10^{-6}$

Figure 50. Chi-square distribution, plot of  $U$  vs  $P_c$   
for various value of  $v$

$P_c$ : cumulative probability

$U$  : variate

$v$  : degree of freedom





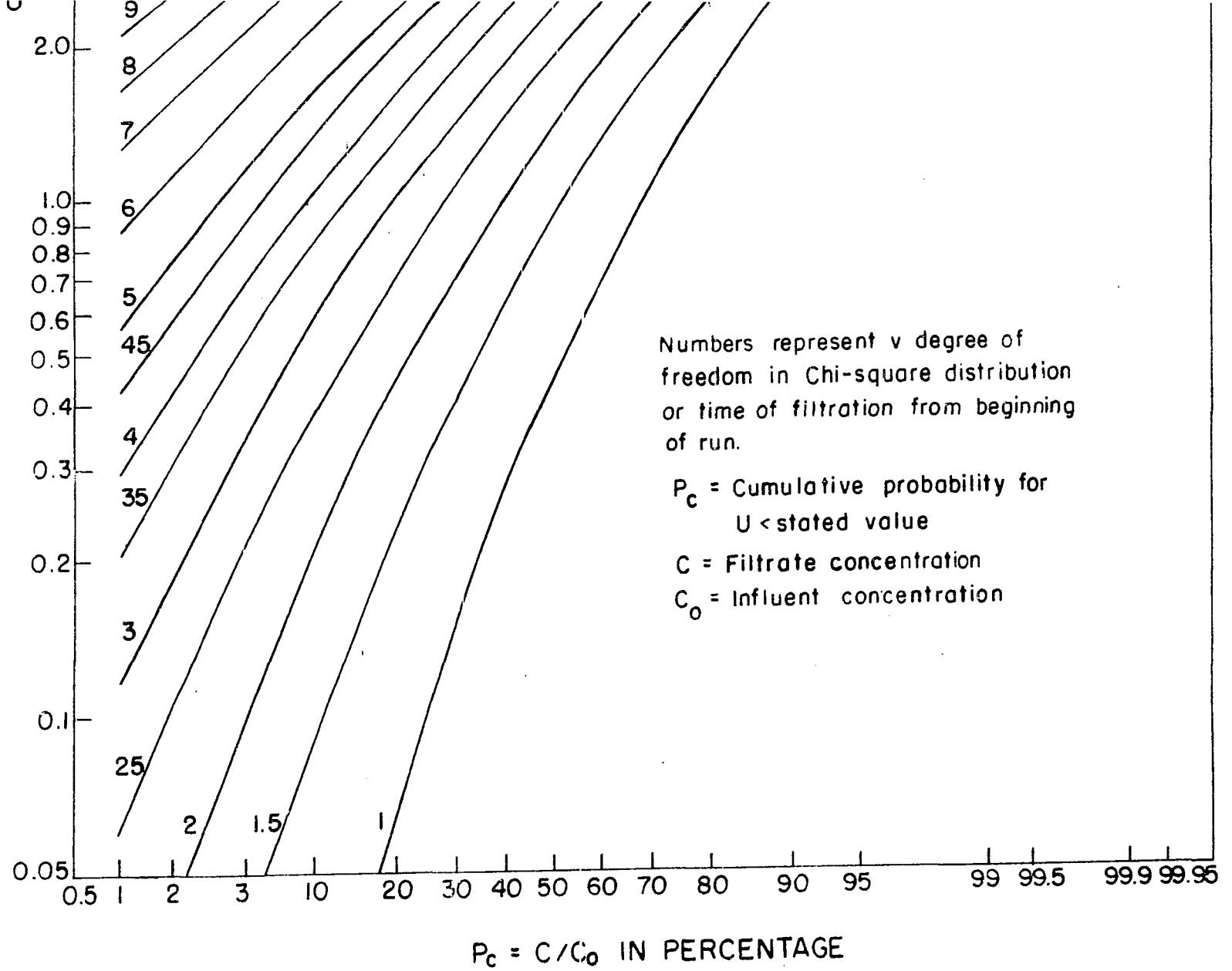


Figure 51. Chi-square distribution, plot of U vs v  
for various values of  $P_c$

$P_c$ : cumulative probability

U : variate

v : degree of freedom

